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Dr. W. W. Coblentz

IVES MEDALIST FOR 1945

AT its New York meeting in October, 1945, the Optical Society of America awarded the Frederic Ives Medal to Dr. W. W. Coblentz, retired physicist of the National Bureau of Standards.*

This medal is awarded biennially for distinguished work in optics, and its past recipients have been as follows:

In 1929, Edward Leamington Nichols;
In 1931, Theodore Lyman;
In 1933, Robert Williams Wood;
In 1935, George Ellery Hale;
In 1937, Herbert Eugene Ives;
In 1939, August Herman Pfund;
In 1941, Selig Hecht;
In 1943, Loyd Ancile Jones.

The presentation of the medal to Dr. Coblentz took place in the Georgian Room of the Hotel Pennsylvania, and was followed by an address by Dr. Coblentz.

Presenting Dr. Coblentz, Mr. William F. Little, Chairman of the Committee appointed to nominate a candidate for the award, spoke as follows:

"The Frederic Ives Medal Committee wishes to report that it has carefully considered its responsibilities and, in its selection of the candidate for the Medal, a complete canvass was made. Since this Medal was last awarded, progress in optics has been great, the knowledge of much of which is still restricted and not generally released. In spite of this restriction, the Committee has found ample evidence of distinguished work by the candidate selected.

"It is with great satisfaction, therefore, that the Committee has recommended Dr. Coblentz as the recipient of this Medal.

"Dr. Coblentz's work has been so important and so voluminous that there is not time to mention more than a few of the outstanding accomplishments.

"The frequency with which his results are used in radiometry and photometry, and the great confidence which all physicists have in him, testifies to his thoroughness and accuracy. It can be said that he established beyond a shadow of a doubt the validity of Planck's law; that the standards of radiation used in this country during the last 30 years have been maintained under his supervision; that modern radiometry from the far infra-red to the extreme ultraviolet is more the creation of one man, Coblentz, than any other equally important branch of physics; and that his 1918 data on the brightness sensitivity of the human eye were among the most important data used in the determination of the standard luminosity curve adopted by the I.C.I. in 1924. These data are still standard today.

"Dr. Coblentz is nearly unique, as a physicist who for 30 or more years has stuck to one difficult and important program of research, radiometry, without being diverted by scientific fads."

* The Ives Medal Committee consisted of William F. Little, David MacAdam, G. W. Moffitt, E. D. Tillyer, and George Wald.

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The Frederic Ives Medal was then presented to Dr. Coblentz by President A. H. Pfund.

On presentation of the medal, Dr. Coblentz replied as follows:

"President Pfund and Fellow Members of The Optical Society of America.

"I wish to express my deep appreciation for the honor which you have bestowed upon me in the form of the Ives Medal.

"Your citation reads: 'For Distinguished Service in Optics.'

"Bestowed as it is after my retirement from the Government Service, this seems more like an '*Ex*tinguished Service Medal,' even though it is difficult to stay *ex*tinguished. Until someone founds a 'Schlemihlung,' where retired researchers can pursue their work to their end, I suppose like Chamisso's mythical 'Peter Schlemihl,' I shall have to wander over the face of the earth in pursuit of my hobbies. Seriously, there is real need for such a laboratory.

"The bestowal of this medal places me in illustrious company, one of whom, Professor Edward L. Nichols, the first Ives Medalist, was largely responsible in starting me out on a career of research; and since recent events show that, in many respects, he was a lone prophet in the wilderness, this is a fitting occasion to recall some of his ideas regarding research.

"Way back, some 41 years ago, at the lunch hour, as we walked across the campus at Cornell University discussing my future vocation, Professor Nichols urged me to devote my life to research.

"In those days there were practically no physics research laboratories as we know them today; and the altruism of the few then extant research laboratories of commercial concerns, in publishing the results of their researches, was then not so common as it is today (as witness the present program of the O.S.A.).

"In my pessimism I replied that one must have a place to work and plenty of worth-while problems. To this, Professor Nichols replied: 'One can always get teachers; but there are no experimenters. As for problems, the solution of one problem brings forth new ones'—Quite a different viewpoint from a recent wail in *Science* that government researchers keep themselves in office by making themselves jobs.

"Then Professor Nichols tóld me of his trip to Washington (attending a meeting of the newly founded American Physical Society) and about the newly organized National Bureau of Standards which promised 'To develop into an institution where scientific work will go on uninterrupted by change of faculty or students; and where work can be done in big projects'—Not, as I have noticed, since then, in one-year spurts to impress a college president to give you a promotion.

"Even in those days the regimenting of scientific work was being discussed. Professor Nichols had the courage to express his convictions on this question. In addressing us, as a member of the Visiting Committee of the National Bureau of Standards, I can still see the earnestness of his countenance as he told us, 'Machine made science is doomed to failure. You cannot contract for the delivery of a discovery on a certain date as mapped out by an advisory board.'

"The pursuit of research problems during the past 40 years has brought me in contact with many scientists in many climes. It was an interesting experience, showing that Professor Nichols was right. The solution of one problem brings forth new ones. What better example do we want than the latest exploits in atomic fission, that open up a new world to the coming generation of researchers in every field of sicence.

"President Pfund, again I thank you and the Members of the Committee on Awards of the Optical Society of America for the honor in bestowing upon me the Ives Medal."

LIST OF SCIENTIFIC PUBLICATIONS OF W. W. COBLENTZ

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(2) "Infra-red emission spectrum of the mercury arc" (with W. C. Geer), Phys. Rev. **16**, 279 (1903).
(3) "Methods of measuring radiant efficiencies" (with E. L. Nichols), Phys. Rev. **17**, 267 (1903).
(4) "Optical properties of iodine," Phys. Rev. **16**, 72 (1903).
(5) "Optical properties of iodine," Phys. Rev. **17**, 51 (1903).
(6) "Selective absorption of fuchsine and cyanine," Phys. Rev. **16**, 119 (1903).
(7) "Selective absorption of organic compounds," Phys. Rev. **16**, 385 (1903).
(8) "Das ultrarote emissionsspektrum des quecksilberlichtbogens" (with W. C. Geer), Physik. Zeits. **4**, 257 (1903).

(9) "Bending of rock salt," Phys. Rev. **16**, 389 (1903).
(10) "Preliminary communication on the infrared absorption spectra of organic compounds," Astrophys. J. **20**, 207 (1904).
(11) "Optical notes: I. Reflection and retraction at the interface of two media having intersecting dispersion curves; II. Infra-red absorption spectrum of selenium," Phys. Rev. **19**, 89 and 94 (1904).
(12) "Investigations of infra-red spectra; Part I. Absorption spectra; Part II. Emission spectra," Pub. No. 35, Carnegie Inst. of Washington (1905).
(13) "Infra-red absorption spectra," Phys. Rev. **20**, 273, 338 (1905).
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(16) "Water of constitution and water of crystallization," Phys. Rev. **20**, 252 (1905).
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(22) "Note on a new form of radiometer," Phys. Rev. **22**, 358 (1906).
(23) "The temperature of the moon," Phys. Rev. **23**, 247 (1906).
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(63) "Note on the reflecting power of tantalum, tungsten, and molybdenum," Phys. Rev. **30**, 645 (1910).

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(65) "Physiologische wirkungen der strahlung," Zeits. f. Beleuch. **16**, 15 (1910).
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(97) "Further experiments on bismuth thermopiles," J. Frank. Inst. **176**, 671 (1913).
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(104) "Zusammenfassender bericht uber versuche mit wismutthermosaulen," Physik. Zeits. **14**, 683 (1913).
(105) "Various modifications of bismuth-silver thermopiles having a continuous absorbing surface," Bull. Bur. Stand. **11**, 131 (1914).
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(111) "Radiation constants of a nitrogen-filled tungsten lamp," Phys. Rev. **4** (February 28, 1914).
(112) "A comparison of stellar radiometers and radiometric measurements on stars," Phys. Rev. **4**, 545 (1914).
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(172) "Spectral energy distribution of the acetylene flame," J. Frank. Inst. **188**, 399 (1919).
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The Measurement of Ultraviolet Radiation Useful in Heliotherapy

W. W. COBLENTZ*

I. INTRODUCTION

THE tribute exacted from the recipient of a medal seems to be a lecture upon some phase of his work. Tonight I shall concentrate my remarks upon the problem of evaluating the biologically effective component of ultraviolet (of wave-lengths 3150A and shorter) in solar and sky radiation, useful for healing purposes.

The background of this task is the establishment of a standard ultraviolet dosage intensity meter, and a unit of dosage, for use in medicine—a problem that was promulgated in 1932 at the Copenhagen Meeting of the International Congress on Actinology ("Lumiere").

At a subsequent meeting of this Congress, in 1936, at Wiesbaden, and in conjunction with the International Meteorological Organization which met at that time in Oxford, England, the specific problem was assigned to a subcommission to investigate a "cadmium" photoelectric cell to measure the biologically effective component of ultraviolet for use in therapy. In the meantime we have passed through a second World War, and at this time nothing is known regarding the progress made in this assignment by workers in different countries.[1]

As the U.S.A. representative on these two Commissions, with the collaboration of R. J. Cashman,[1-3] I was fortunate in developing a new type of photoelectric cell (Fig. 1) for measuring ultraviolet solar and sky radiation incident on a horizontal plane. In the early part of the development of this type of photoelectric cell, I was aided by two grants from the Council on Physical Medicine of the A.M.A. (of which I am a member) and a subsequent grant from the American Philosophical Society. Unfortunately, aside from the collaboration of R. Stair[3,4] in making a primary calibration of several of these instruments in absolute units, because of the military exigency, throughout the past 4½ years it fell to my lot to service the various instruments that I have studied (about 2 dozen in all) and maintain the primary standard. Although provided with automatic integrating and recording apparatus, the servicing of these ultraviolet meters has been an onerous task, because it has not yet been possible to produce photoelectric cells that do not require stabilization in sensitivity, generally by exposure to the sun and weather. Progress was also impeded by my inability to get deliveries of new types of photoelectric cells.

With all these delays and difficulties in trying to bring to conclusion my contribution to the task assigned by the above-mentioned International Commissions, I am deeply grateful to the National Bureau of Standards for the opportunity to continue (as its guest) my study of instruments and methods of measuring biologically effective ultraviolet radiation for use in medicine.

The whole problem of evaluating ultraviolet in sunlight arose from the observation that ordinary

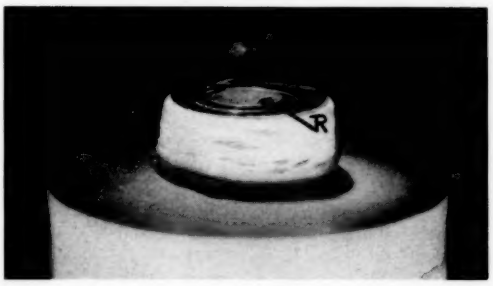

FIG. 1. Top part of metal housing showing photo-sensitive receiver, *R*, of Zr-photo-tube, and outer heavy wire ring that supports the bowed, positive electrode of fine wires.

* Address delivered by the recipient of the Frederic Ives Medal on the occasion of its presentation at the Thirtieth Annual Meeting of the Optical Society of America, New York City, on October 19, 1945.

[1] W. W. Coblentz, "Bioclimatic measurements of ultraviolet solar and sky radiation in Washington, D. C., 1941–1944," Bull. Am. Meteorolog. Soc. **26**, 113 (1945). In this paper see footnote reference (1) to members of International Commissions.

[2] W. W. Coblentz and R. J. Cashman, "A photoelectric cell for measuring ultraviolet solar and sky radiation on a horizontal plane," Bull. Am. Meteorolog. Soc. **21**, 149 (1940).

[3] W. W. Coblentz and R. Stair, "A daily record of ultraviolet solar and sky radiation in Washington, D. C., 1941–1943," J. Research Nat. Bur. Stand. **33**, 21 (1944). In this paper see reference 1 for acknowledgments.

[4] W. W. Coblentz, R. F. Gracely, and R. Stair, "Measurements of ultraviolet solar and sky radiation intensities in high latitudes," J. Research Nat. Bur. Stand. **28**, 581 (1942).

window glass shuts out ultraviolet of short wave-lengths (2900A to 3200A) in sunlight that prevents (and cures) rickets. My interest in this problem began about two decades ago with the high pressure sales promotion of special window glasses, transparent to this band of ultraviolet radiation, for use in solariums, schools, nurseries, etc.

In order to determine the probable benefits to be obtained by using special window glasses, it was necessary to evaluate the energy in the spectral band of solar radiation extending from the limit of transparency of the earth's atmosphere (at about 2900A) to about 3200A, which, as already stated, is specific in curing rickets, causing sunburn, producing vitamin D, etc. Such an evaluation, in absolute units, requires a knowledge of the distribution of energy in this spectral band.

As is well known, in addition to the Fraunhofer absorption lines, which originate in the sun's atmosphere, this spectral region is filled with absorption bands caused by the presence of ozone in the earth's atmosphere.[5] It is therefore impossible to procure a spectral energy curve of solar radiation that is smooth and free from indentations. Moreover, for a sea-level station no one has yet observed the solar spectral energy distribution in these short wave-lengths. In fact, for large air masses traversed by the solar rays, such measurements would have but little meaning.

The question was how the promoters of special window glasses obtained their extravagant values of high transparency of their particular brands of window glass to antirachitic solar radiation. My surmise was that they guessed at the shape of the spectral energy curve in the region of 2900A to 3200A. And that is exactly what we did in those early tests some 18 years ago. But our (guesses), "cut-to-fit" estimates of the shape of the spectral energy curve at different solar heights, are based upon scientific principles.

By means of a photoelectric cell that responds only to ultraviolet radiation of wave-lengths shorter than about 3400A, and a set of 4 glass filters having a sharp ultraviolet cut-off, the spectral band of radiation extending from 2900A to about 3400A is separated into 5 parts.

In this manner the amount of integrated ultraviolet, transmitted by these filters, is observed at different solar heights, i.e., air masses traversed by solar radiation, incident normally upon the photo-cell.[5] Then from (1) the known spectral transmissions of the glass filters, (2) the known spectral response of the photoelectric cell, and (3) an assumed spectral energy distribution for a given solar height (say, air-mass $m = 1.1$ traversed by the solar rays) the calculated integrated percent transmissions are compared with the observed filter transmissions. By changing the assumed intensities in the region of, say, 2900A to 3100A relative to the intensities at 3200A to 3500A, a closer agreement is obtained between the calculated and the observed amount of integrated ultraviolet solar radiation transmitted by these filters.

I shall always remember the thrill in starting with Pettit's ultraviolet solar spectral energy measurements, observed at Tucson, Arizona (elevation about 2700 ft.), and on increasing the relative spectral intensities at 2900A to 3000A to fit the estimated conditions at my observing station at the Lowell Observatory, Flagstaff, Arizona (elevation 7300 ft.), on the third trial I obtained calculated integrated spectral transmissions that were in agreement, within 0.2 to 0.3 percent, with the transmissions observed with four filters and two photoelectric cells differing appreciably in spectral photoelectric response.

While this "cut-and-try" method became our standard procedure, never after that first day's observations was I able, with so few trials, to estimate the shape of the spectral energy curve to give calculated integrated filter transmissions that were so close to the observed values. The reason is obvious. As shown in our published papers,[5, 6] for the same solar heights, atmospheric conditions are never exactly the same on different days. Hence we calculated data and prepared graphs of integrated filter transmissions for spectral energy distributions representative of solar radiation after passing through air-masses, $m = 1.0, 1.1, 1.2, 1.5, 2.0, 2.5$, etc., under standard

[5] W. W. Coblentz and R. Stair, "Factors affecting ultraviolet solar-radiation intensities," J. Research Nat. Bur. Stand. 15, 124 (1935).

[6] W. W. Coblentz and R. Stair, "Evaluation of ultraviolet solar radiation of short wavelengths," J. Research Nat. Bur. Stand. 16, 316 (1936).

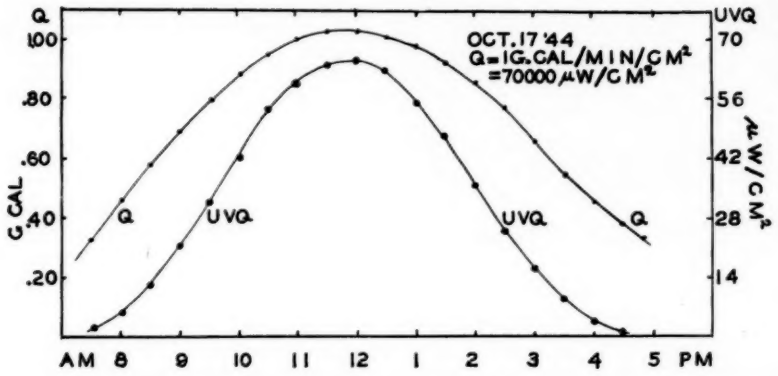

FIG. 2. Comparison of the diurnal variation in the total radiation, *Q*, of all wavelengths and of the biologically effective ultraviolet component, *UVQ*, radiation from the sun and the whole sky incident on a horizontal plane on a clear day in Washington, D. C.

conditions.[6] On especially clear days the observed integrated filter transmissions may correspond with air-masses that are smaller than the values deduced from the observed solar heights. In the primary calibration of ultraviolet intensity meters, in absolute value, which includes simultaneous measurements of intensities of ultraviolet solar radiation at normal incidence, these deviations in spectral energy distribution are taken into consideration.

Since the photoelectric cell magnifies the low intensities at 2900A relative to the longer wavelengths, and since the filter measurements require only a few minutes at the time of the standardization of the dosage intensity meter, I consider the photoelectric cell and filter method more practical and just as reliable as the slow, tedious, spectroradiometric method of obtaining the shape of the solar spectral energy curve in the shortest ultraviolet region of the spectrum.

II. APPARATUS AND METHODS

For a complete description of the photoelectric cell, methods of standardization of the intensity meter in absolute units, methods of making the measurements, and a summary of observations extending over a period of some 4 years, reference is made to previous papers.[3–6]

It will suffice to say that the ultraviolet intensity meter consists of a photoelectric cell with a flat circular receiver that is exposed horizontally to intercept radiation incident from the sun and the whole sky.[2]

The electrons released from the photosensitive surface of the receiver charge a small condenser, which, on becoming charged to a critical potential, discharges through a 3-electrode "trigger tube."[7] This electric discharge ("impulse") operates an ordinary message counter. The frequency of the impulses is a measure of the ultraviolet intensity. The combination is an excellent integrating device that (by means of an electric clock) operates from dawn till dusk. In addition the writer uses an electrically operated impulse ("traffic") counter that prints upon a strip of paper, the day of the week, the hour of the day, and a summation of the impulses at half-hour or quarter-hour intervals. It is, therefore, a simple matter to determine the total number of impulses per day, etc.

Knowing the energy required to charge the condenser to produce one impulse ("click" in microwatt seconds, per cm²: μ wsec./cm²), it is a simple matter to obtain a record of the intensity at any given time, and the total amount of biologically effective ultraviolet solar and sky radiation incident at a given place.

For example, using a certain ultraviolet intensity meter having a calibration factor of 1 "click" (impulse) = 1350μ wsec./cm², at the noon hour on a clear day in June, the meter operated at a rate of 1 impulse in 9 seconds, indicating a dosage intensity of $(1350 \div 9 =)$ 150μ w/cm².

[7] H. C. Rentschler, "An ultraviolet light meter," Proc. A. I. E. E. **49**, 576 (1930); Trans. Am. Illum. Eng. Soc. **25**, 406 (1930).

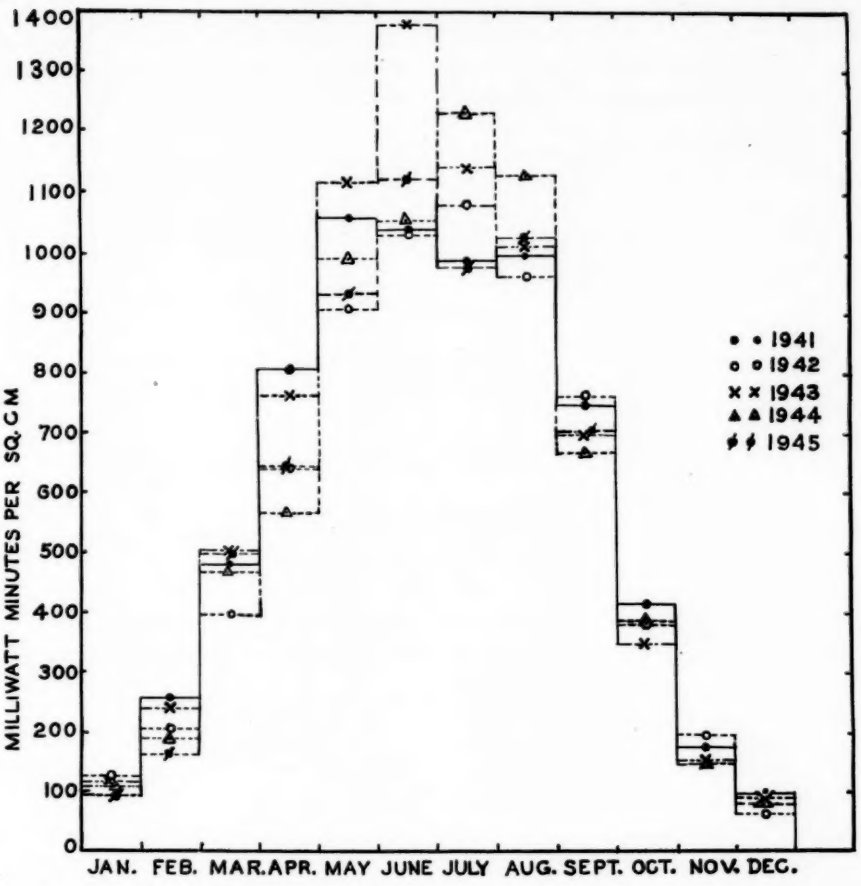

FIG. 3. Monthly totals of biologically effective ultraviolet solar and sky radiation in milliwatt minutes per square centimeter incident on a horizontal plane in Washington, D. C., February 1, 1941 to September 1945.

This is sufficient to produce an erythema on the untanned skin in 15 to 20 minutes.

Knowing the calibration factor of his ultraviolet intensity meter, by counting the total number of impulses,[3] the physician can determine the total dose of ultraviolet (a difficult task with passing clouds) that the patient is receiving, just as he prescribes grains of aspirin and drops of paregoric.

In Fig. 2 is illustrated the marked depletion of the biologically effective ultraviolet component relative to the total incident solar and sky radiation when the sun is low on the horizon. On this particular date (October 17, 1944; a clear sky) the total intensity incident on a horizontal plane, as measured with an Eppley thermoelectric pyrheliometer, was slightly more than 1.0 g cal./min./cm^2, or 70,000μ w/cm^2. The biologically effective ultraviolet component, as measured with a photoelectric ultraviolet intensity meter, was less than 1/1000 of this value (which is the average value for a sea-level station) at the noon hour; and it is practically zero at 4:30 P.M., when the total radiation of all wave-lengths was still close to 0.4 g cal./min./cm^2, or practically $\frac{1}{2}$ of the noon-hour intensity.

In Fig. 3 is shown a record of monthly totals of biologically effective ultraviolet solar and sky radiation, in milliwatt minutes per cm^2, incident on a horizontal plane in Washington, D. C.,

during the period of February 1, 1941 to September 1, 1945.

In this connection it is of interest to note that the lowest value recorded in July, 1945, corresponds with the greatest precipitation ever recorded in Washington during this month; whereas the month of August, 1945 (with only ½ the average precipitation), had relatively less cloudy weather, and close to the monthly average ultraviolet solar radiation observed during the 5-year period.

Obviously, once we have a primary standard ultraviolet intensity meter to be maintained at the National Bureau of Standards, it will be an easy task to calibrate others that can be taken to distant stations for bioclimatological and photochemical investigations. In fact, such photochemical investigations are already in progress. The great need is photoelectric cells that remain constant in sensitivity. It is to be hoped that the investigation of this question can be resumed in the near future.

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Visual Critical Flicker Frequency as a Function of Intensity*

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(Received November 13, 1945)

IN vision of man and of most vertebrates the critical flicker or fusion frequency (F) with white light as a function of the logarithm of the intensity (I) is a double sigmoid curve consisting of a low intensity or rod portion and a high intensity or cone portion. If the wave-length of the stimulating light is 670 mμ or longer the rod section is absent, and the curve has a simple sigmoid shape (Hecht, Shlaer, and Smith,[25] Hecht and Smith,[26] Hecht and Shlaer,[24] Crozier and Wolf[12]). In vertebrates which possess only rods or cones or in arthropods which possess only one type of visual sense cell, the $F-\log I$ curve likewise is of a simple shape (Wolf,[31] Crozier and Wolf,[6,9,10] Crozier, Wolf, and Zerrahn-Wolf,[19,20] and others).

Two theories have been proposed to explain the shape of the curves: (1) the photochemical theory as formulated by Hecht, Shlaer, and Smith,[25] and (2) the theory of a wide distribution in thresholds (four to eight log units) so that the sigmoid $F-\log I$ curves become probability integrals (Crozier,[3] Crozier, Wolf, and Zerrahn-Wolf,[16] and other papers cited above). Since neither theory is entirely satisfactory, it is the purpose of the present paper to propose a reformulation of the photochemical theory.

* Aided by grant from the Rockefeller Foundation for research on the physiology of the normal cell.

THE OLD EQUATION

It is generally assumed that, in accordance with the theory of photochemical control of flicker fusion, the critical flicker frequency should vary with the concentration of photo-products in the photo-receptor cells (Hecht and Verrijp,[27] Hecht, Shlaer and Smith,[25] Hecht,[21] Crozier, Wolf, and Zerrahn-Wolf.[13,4]

According to the photochemical scheme of Hecht[21,22] a photo-sensitive substance, S, is converted reversibly into the photo-products, P and A. If a is taken to denote the total available concentration of photo-sensitive substance and x the amount which is in the form of photo-products, the following equations for the kinetics of the reactions may be derived. During the illuminated phase of the flicker cycle

$$dx/dt = k_1 I(a-x) - k_2 x^2, \qquad (1)$$

where k_1 and k_2 are velocity constants for the light and dark reactions, respectively. During the dark phase

$$-dx/dt = k_2 x^2. \qquad (2)$$

If $1/p$ is used to denote the fraction of the time

during which the light is on, then the equilibrium equation for the pseudo-steady state at the critical flicker frequency becomes

$$k_1 I/k_2 p = x^2/(a-x). \tag{3}$$

This is the general equation for the Talbot law (Hecht and Wolf,[28] Hecht and Verrijp,[27] Hecht[21]).

In the early papers by Hecht on the photochemical cycle it was pointed out that at least one of the photo-products (P or A) acts as a catalyst for a reaction in which an indifferent material, L, is converted into an active agent, T, which causes stimulation of the nerve endings. This reaction has been written as

$$L\|P+A\|\rightarrow T \tag{4}$$

and is usually referred to as the "secondary" dark reaction. In order to explain the cessation of stimulation with cessation of illumination it is necessary to assume that the activating material (P or A, or both) does not remain active but is rapidly converted into an inactive form from whence it may be rebuilt into S. Only the "new" $P+A$ is a catalyst.

According to Hecht[21] an equation for critical flicker frequency may be derived as follows:

"The effect of light on the photo-sensitive substance is carried forward by the secondary dark reaction which the photo-products undergo. According to Eq. (3) above the concentration of these photo-products is x; therefore, the rate of the secondary dark reaction is

$$(\Delta x/\Delta t)_{\text{secondary}} = k_3 x, \tag{5}$$

where k_3 is a velocity constant. For the critical disappearance of flicker suppose that $\Delta x = c'$; that is, that the change Δx in concentration in the dark time Δt is just too small to cause the physiological change corresponding to a change in sensation of brightness. . . . The critical fusion frequency is F cycles of light and dark flashes per second; thus $F = 1/2\Delta t$. Substituting these values of Δx and Δt in Eq. (5) we get

$$F = cx, \tag{6}$$

where $c = k_3/2c'$. In other words, the critical fusion frequency is directly proportional to the concentration of photo-products in the stationary state as given by Eq. (3) above."

An earlier equation,[27]

$$F = x^2/c^2 \tag{7}$$

has been abandoned.[24]

Since according to (6) F is a linear function of x, it may be substituted for x in the equilibrium equation and $F_{\max}$ may be substituted for a. Equation (3) then becomes

$$\frac{k_1 I}{k_2 p} = \frac{F^n}{(F_{\max}-F)^m}, \tag{8}$$

or

$$F = \left[\frac{k_1 I}{k_2 p}(F_{\max}-F)^m\right]^{1/n}, \tag{9}$$

or, by changing the absolute value of k_2,

$$F = \frac{1}{pk_2}(k_1 I)^{1/n}(F_{\max}-F)^{m/n}, \tag{10}$$

where m and n denote the order of reactions $S\rightarrow P+A$ and $P+A\rightarrow S$, respectively.

DEFICIENCIES OF THE OLD EQUATION

Several difficulties have arisen in the use of Eq. (10) as an explanation of the $F-\log I$ curves. They are as follows:

(1) With change in temperature the predicted shift along the I axis is in the direction opposite to that which occurs (Crozier, Wolf, and Zerrahn-Wolf[14,15,19]; Crozier and Wolf,[5,7] and earlier papers).

(2) With change of the light to dark ratio the predicted shift is opposite to that which occurs (Bartley,[1,2] Crozier, Wolf, and Zerrahn-Wolf,[17,18] Crozier and Wolf[8,11,12]).

(3) In order to fit the data on human vision it is necessary to assume that the photochemical reaction is bimolecular. This assumption is contrary to the available chemical evidence.[23]

(4) For certain animals the flicker data (especially for *Pseudemys* and *Sphaerodactylus*, and the low intensity data for *Asellus*) cannot be fitted with Eq. (5) (Crozier and Wolf,[4] Crozier, Wolf, and Zerrahn-Wolf[19,20]).

(5) On purely logical grounds it seems probable that the concentration of S rather than the concentration of $P+A$, should determine directly the critical flicker frequency. It is generally assumed that the effect (and/or the amount) of "new" $P+A$ necessary for stimulation at

threshold is the same for all intensities and frequencies. The amount of light necessary for a given small photochemical effect is inversely proportional to the concentration of S. Therefore, if the amount of photo-sensitive substance is increased (e.g., by a rise in temperature or an increase in the dark time) the critical flicker frequency at a given intensity should be higher, and the intensity for a given value of the critical flicker frequency should be lower. This is exactly what happens.

The formulation of Hecht seems to be open to criticism principally because the concentration of the active catalyst ("new" $P+A$) which is the direct cause of stimulation is not taken into consideration. Since the "old" $P+A$ which is represented by x has no active part in the production of a stimulus, the possibility exists that the observed correlation between F and x is spurious. The difficulties listed above make Eqs. (8)–(10) untenable.

THE NEW EQUATION

It is the purpose of the present paper to attempt a new photochemical explanation of the shape of the $F-\log I$ curves which will not be subject to the above deficiencies. This explanation is based on two main assumptions; (1) That F varies as the reciprocal of the time of the flash necessary to produce a given critical catalytic effect of "new" $P+A$ which is "just too small to cause the physiological change corresponding to a change in the sensation of brightness." [21] (2) That the maximum of F is determined not by the photochemical changes in the sense cell but by some other factor and that this other factor is not affected by temperature.

Since P and A are commonly used to denote "old" or inactive materials, let us designate the active material by E and its concentration by w. The photochemical cycle then becomes

$$S \rightarrow E \rightarrow P+A \rightarrow S \cdots.$$

In view of recent chemical research[23] we might consider E to be transient orange and $P+A$ to be visual yellow. The reaction which results in stimulation of the nerve is determined by the concentration of E, but the kinetics of dark adaptation are determined by the concentration of P and A.

Let us now derive an equation for the critical effect of E as a function of intensity.

The amount of E formed should vary with the intensity (I), with the amount of S present, i.e., $[S]$ or $(a-x)$, and with the duration of the flash (t). The amount formed by a single flash, then, is $k_1 I(a-x)t$. It is assumed that E is small compared to $a-x$ (except as stated below) which may be calculated from Eq. (3). The secondary reaction ($L \rightarrow T$) depends upon the concentration of E, the concentration of L (which will be designated by l), and the velocity constant (k_4) of the catalyzed reaction. The rate of this reaction is, therefore, $k_4 lw$ and the quantity of L converted to T is approximated by $k_4 lwt$. We could assume for conditions of critical flicker either of these quantities is constant, i.e., either $k_4 lw = C$ or $k_4 lwt = C$. We shall be able to superimpose $F-\log I$ curves calculated on these two assumptions by changing the values of k_4, and of C, and q, two constants to be discussed below. Since the $L \rightarrow T$ reaction is either reversible or cyclic, let us assume that the rate of this reaction must reach some threshold value, i.e., $k_4 lw = C$. Therefore,

$$C = k_4 l[k_1 I(a-x)]t. \tag{11}$$

For any given value of p the critical rate of flicker (F) should vary with the reciprocal of the time necessary to produce this constant physiological effect. Therefore, $F \propto 1/t$ and

$$\frac{1}{t} = \frac{k_4 l}{C}[k_1 I(a-x)]. \tag{12}$$

If the $L \rightarrow T$ reaction is regarded as any of the chemical and electrical changes which normally result in stimulation of the nerve[32] then it seems reasonable to assume that the available amount (or concentration) of L undergoes considerable variation during a flicker cycle, decreasing when the light is on and increasing when it is off. At the beginning of a flash the available material (l), therefore, varies as a function of the time in the dark, i.e., $l = f(p)$. As a simple first approximation, which will probably need to be changed as more data become available, we may consider l to be a linear function of p. Therefore, we may temporarily place p in Eq. (12) in place of l and include the proportionally constant in C, which thereby becomes C'. Actually, l should be re-

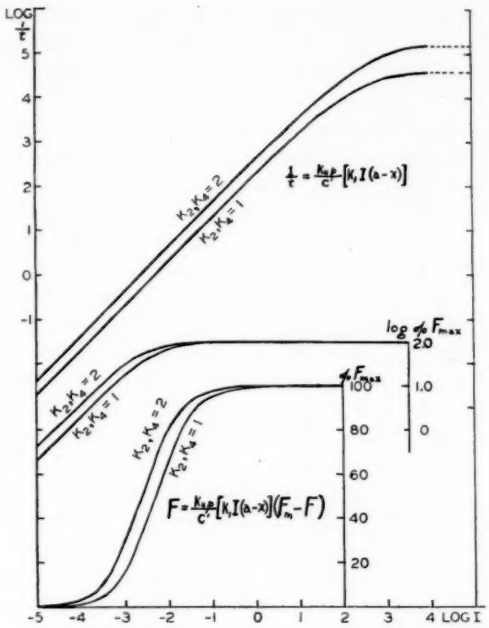

FIG. 1. Upper pair of curves: $1/t$ as a function of log I, according to Eq. (12). Middle pair of curves: log F as a function of log I according to Eq. (15). Lower pair of curves: F as a function of log I according to Eq. (15).

placed by some average value of $[L]$, which, however, should also vary as p.

$$\frac{1}{t} = \frac{k_4 p}{C'}[k_1 I(a-x)]. \qquad (13)$$

If for Eq. (13) calculated values of $1/t$ are plotted against log I, a sigmoid curve results; and this curve shifts with temperature (increase of k_2 and k_4) in such a way that the value of $1/t$ for any given value of I also increases (Fig. 1). Therefore, one might be tempted to consider $F = 1/t$. However, the maximum value of $1/t$ is also increased by an increase in k_2 and k_4. Therefore, the second principal assumption above, i.e., that the maximum is determined independently of the photochemical theory, is made. In general terms we may state simply that $F \propto f(F)$ and may write the equation for flicker frequency as

$$F = \frac{k_4 p}{C'}[k_1 I(a-x)] \cdot f(F). \qquad (14)$$

This $f(F)$ does not pertain to the photochemical mechanism by which the sense cell is stimulated but to the mechanism whereby the sensation of flicker approaches a maximum when the frequency of repetitive equivalent stimuli is increased. The controlling process which governs this phenomenon may lie any place in the catenary series of events between the reaction denoted by Eq. (4) and the sensation of flicker. Since this mechanism is undetermined, we may consider $f(F)$ to be any function which will fit the data. One simple expression for this function is $(F_{max} - F)^r$. Therefore, one form of Eq. (14) is

$$F = \frac{k_4 p}{C'}[k_1 I(a-x)]^q (F_{max} - F)^r, \qquad (15)$$

where we let q denote some power of the bracketed expression to be discussed later. F_{max} is assumed to be almost independent of temperature, and, therefore, may be controlled by some physical rather than chemical process.

Curves for this equation are plotted in Fig. 1. The chosen values of the constants were $p = 2$, $q = 1$, $r = 1$, $k_1 = 1$, $C' = 1$, $a = 100$, $F_{max} = 100$ for all curves, with k_2 and $k_4 = 1$ in one curve and $= 2$ in the other. The constants k_2 and k_4 are for thermal reactions, and the increase in value, therefore, represents an increase in temperature. The shift in position is in the direction of the available experimental data.[5, 7, 14, 15, 19]

The curvature in the bend of the log−log graph can be decreased slightly by assuming that all reactions are monomolecular. The curve can be shifted on the log I axis by assuming different values for C' or k_4, and this also tends to change the curvature at the upper end, especially if C' is increased by a factor of 10 or more. The curvature of this bend could be changed more effectively by changing the value of q or r. The constant q determines the low intensity terminal slope.

At very high intensities the amount of S does not remain approximately constant during a flash cycle, i.e., the amount at the beginning of the flash is appreciably greater than at the end of the flash. (A similar change in S during a flash cycle for low frequencies has been discussed previously in connection with brightness enhancement.[29]) Therefore, the expression for w becomes undependable. In Fig. 1 the bracketed expression of Eq. (15) is plotted as a horizontal

dotted line because this seems as good an approximation as may be obtained by relatively simple mathematics. In this section of the curve the factor which controls $1/t$ is not so much I or $(a-x)$ as it is the reaction $P+A \rightarrow S$; and in order to introduce this reaction into the new equation, it seems necessary to set up three differential equations[30] (see below) or at least to introduce into Eq. (15) a complex factor for the absolute value of the dark time which would be operative only when $[S]$ became very small. However, if the temperature is raised, the velocity of the $P+A \rightarrow S$ reaction is increased, and the $F-\log I$ curve should shift to the left, as it does in experimental data.

It can be noted from Fig. 1 that the quantity

$$[k_1 I(a-x)]/C$$

is a linear function of I from the extreme low range to $F=$approximately 99 or $\log I=1$. Therefore, we may substitute I for this quantity in Eq. (15) which becomes

$$F=\frac{k_4 p}{C''}[I]^q(F_{\max}-F)^r, \qquad (16)$$

where C'' includes the necessary proportionality constant, and Eq. (15) is thereby reduced to the form of Eq. (10). The present equation, therefore, can be used to fit all data which could be fit by Hecht's equation, particularly that for human rod and cone vision. For the cone curve, because of the change in q, the substitution of I for the bracketed expression is not quite so exact as for the rod section. However, the introduced error is negligible below $F=90$ percent, 0.07 log unit at $F=95$ percent, and about 0.5 log unit at $F=99$ percent. Since the curve is almost horizontal at 99 percent, even this shift of 0.5 log unit is negligible, and one curve fits the data as well as the other, especially since experimental errors are large when F is almost equal to $F_{\max}$. If any adjustment in this range should become necessary, it could be obtained by changing the value of C' and therefore of C''. This constant controls the extent of the non-linear section of the

$$[k_1 I(a-x)]k_4/C'-\log I$$

curve which enters into Eq. (15).

Although the new (16) and the old (10) equations are identical in form the parameters have quite a different significance. In the exponent m/n or r instead of m being the order of the $S \rightarrow P+A$ reaction, r denotes the $f(F)$ discussed above. In the exponent $1/n$ or q instead of n being the order of the reverse reaction, q probably pertains to the mechanism whereby the photo-product E is utilized by the sense cells. Since this mechanism is undetermined the necessary value of $\frac{1}{2}$ for q (or 2 for n) necessary to fit the cone data does not seem implausible.

The difference in the significance of the parameters of the old and new equations permits much greater flexibility in the fitting of the equation to experimental data. In the old equation m and n could be only small integers, namely 1 or 2, since they indicated the order of the reactions. Since, on the basis of chemical investigation, m was known to be 1, this left only two possible shapes for the $\log F-\log I$ curves. In the new equation q and r may, at least for the present, be given any value (whole or fractional) which will fit the data and these values, especially r, might conceivably change with temperature. Changes in the values of C' and also (if it seems necessary and permissible) in the order of the reactions in Eqs. (1) to (3) offer further possibilities of adjustments.

In spite of the close similarity in form of the old and new equations there are two notable differences between them. One of these differences is in the position of the velocity constants which determine the position of the curve on the $\log I$ axis. In both equations the constant k_1 does not change with temperature, and k_2 and k_4 do. Therefore, in the new equation where $F \propto k_4$, the curve will shift with temperature in a direction opposite that of the old equation where $F \propto 1/k_2$. One major criticism of the old theory was that the predicted shift with temperature was contrary to fact. In the new equation the theory predicts the direction of the observed shift, and criticism of the photochemical theory on this basis is thereby removed. From the magnitude of the shift the change in k_2 and k_4 could be calculated if one of these were known or assumed. Since the shift for a 10°C change is in all known cases considerably less than one log unit (which corresponds roughly to a doubling of k_4) it seems probable that k_4 should be placed inside the bracket in Eq. (15), a modification from which

a smaller shift would be predictable if q were less than one. This position for k_4 is just as logical as that given above. If the reversibility of the $L \rightarrow T$ reaction were considered in the above equations it might also reduce the magnitude of the predicted temperature shift.

The other main difference between the new and old equations is in the position of p. In Eq. (10) $F \propto 1/p$, and this is contradictory to experimental data. In Eq. (15) p affects any submaximal value of F in two ways: 1) $F \propto p$ because p is a factor in the bracketed expression, 2) $F \propto (a-x)$ which in turn varies as a function of p (Eq. (3)). The first relationship is effective over the entire intensity range, and the second only at high intensities where the substitution of I for the bracketed expression in Eq. (15) is not valid. Therefore, if p is increased, for any given value of I, F will be higher. That is what occurs when p is changed experimentally. The exact form of the curve for man varies with intensity,[1,2] but the general trend is for F to vary as p.

It has also been found[17] that F_{max} increases in proportion to the percentage of the dark time. Let us consider what happens at very high intensity where $a-x$ is very small and varies considerably during the flash cycle. At the beginning of the cycle the amount present will vary with the length of the dark time, and therefore, if other conditions permit, F should vary directly with the dark time. This possibility is not necessarily valid because we have assumed that the maximum value of F is determined by processes other than those in the sense cells. However, even if we maintain this assumption, an increase of F_{max} seems possible and predictable. F_{max}, presumably, is controlled by some recovery process the extent of which is affected by the time in the dark. Therefore, an increase of the dark time should result in an increase in the degree of recovery possible. Consequently, for the same degree of recovery an increase in the percentage of the dark time (accompanied by an increase in intensity) should permit a higher value of F_{max}, such as has been observed experimentally.

If one were going to be thoroughly logical about the application of the photochemical theory it would be necessary to set up equations for the complete course of the $S \rightarrow E \leftrightarrow P+A$ reaction during each cycle of flash.[30] For the period of illumination the net rate of formation is

$$[k_1 I(a-w-x)-k_3 w].$$

If the integral of this expression with respect to t (the quantity which is approximated by w on page 78) is multiplied by $k_4 l$ and the product set equal to a constant (equivalent of $k_4 l w = C$, above), it yields a function for $1/t$ which varies approximately as I throughout most of its range.

$$\frac{1}{t} = -(k_1 I + k_2) \Big/ \ln\left(1 - \frac{C''(k_1 I + k_2)}{k_4 l k_1 I(a-w-x)}\right). \quad (17)$$

The curve for $1/t$ according to this equation is similar to that for $1/t$ in (12). The integrated expression is more complex than the bracketed expression of (12), offers no immediate advantages, and is subject to the same inaccuracies as (12) when $[S]$ becomes very small (see above). If one wishes to consider the possibility that $k_4 l w t = C$ on the basis of the integral expressions, C should be placed equal to the second integrals of the sum of the net rate of formation and the rate of removal (during dark period) of E. This assumption leads to an equation for $1/t$ which is algebraically insoluble and at present does not seem worthy of further investigation.

Crozier and co-workers have pointed out that the data on *Pseudemys*, *Sphaerodactylus*, and that for the low intensity range on *Asellus* can not be well fitted with Eq. (8). However, because of the great flexibility of the new equation it can be fitted reasonably well to all of the available experimental data. For example, the data for *Pseudemys* (reference 19, p. 314) can be fitted by Eq. (15) if $q = 1/1.75$ and $r = 8$ (Fig. 2). This is equivalent to making $m = 8n$ and $n = 1.75$ in Eq. (10), which would be impossible on the basis of the old theory.

The new equations ((14) and (15)) are not proposed as a final solution for the explanation of flicker data. However, they do remove all of the objections to the photochemical theory as the basis of such an explanation. The particular significance of the exponent q and the form of $f(F)$ is left unsolved. According to the present interpretation these may differ in different types of visual cells and/or different species of animals, and the exact significance of these factors is

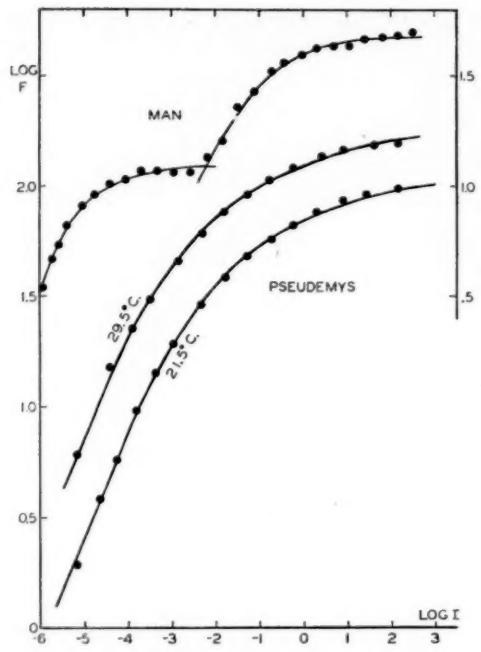

FIG. 2. Upper curve, scale on the right: log F as a function of log I, data of Hecht and Smith (1936), curve calculated from Eq. (16) with $q=1$ for rods and 0.5 for the cones, and $r=1$ for both rods and cones.

Lower curves, scale on the left: log F as a function of log I, data of Crozier, Wolf, and Zerrahn-Wolf (reference 19) for *Pseudemys*. Data for 29.5°C has been shifted upward by 0.2 log unit, curves were calculated from Eq. (15) with $q=1/1.75$ and $r=8$.

beyond the scope of the present paper. The exact form of the relationship $I=f(p)$ is also left for future investigation.

SUMMARY

An equation for visual critical flicker or fusion frequency as a function of intensity has been derived on the basis of the photochemical theory. It is assumed that the flicker frequency is a function of the concentration of photosensitive substance and also of the manner in which the maximal value is approached as the frequency of equivalent stimuli is increased. This equation has the same general form as the equation of Hecht, and, therefore, will fit all of the data which could be fitted by the old equation. However, the parameters of the two equations have quite a different significance. Because of this change in significance of the parameters the flexibility of the new equation allows much more freedom in adjusting the equation to fit experimental data, and all known flicker data can, therefore, be fitted reasonably well. According to the new equation a rise in temperature should lower the intensity at which fusion occurs. This is in agreement with experimental data and contrary to predictions on the basis of the old equation. The new equation also predicts the observed shift in the flicker-intensity curves when the ratio of the light to dark period is changed, and it makes no assumptions regarding the photochemical cycle which contradict the chemical evidence.

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Brightness Discrimination and Visual Acuity as Functions of Intensity*

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HECHT[8–11,13] and Steinhardt[24] have demonstrated that the relationship between brightness discrimination and intensity and also between visual acuity and intensity can be described by the equilibrium equation for the photochemical cycle. This equation is

$$kI=x^n/(a-x)^m. \tag{1}$$

The ratio of the just perceptible change in illumination (ΔI) to the prevailing illumination (I) is inversely proportional to the nth power of the amount of photo-product, i.e.,

$$\Delta I/I \propto 1/x^n.$$

For data on human vision the exact form of the equation was

$$\Delta I/I=C[1+1/(kI)^{\frac{1}{2}}]^2. \tag{2}$$

A modification of this equation was made by Bartlett[1] and Bartlett and Hudson.[2] Alternative theories have been proposed by Crozier and Holway,[5] by Householder,[17] by Householder and Landahl,[18] and by Moon and Spencer.[20]

Hecht and Mintz[15] have demonstrated that visual acuity is a form of brightness discrimination in the region of the diffracted retinal image and that the critical visual angle α is directly proportional to $\Delta I/I$. Therefore, any function of intensity which fits the data for brightness discrimination will also fit that for visual acuity. An elaboration of the photochemical theory as applied to visual acuity has been made by Byram[4] in terms of the energy rates of the reaction catalyzed by the photo-product. Thorough discussions of various phases of the problem of visual acuity have been published by Walls[25] and Byram.[3,4]

In order to fit the data for both brightness discrimination and visual acuity with Eq. (1) it is necessary to assume $m=n=2$. The assumption of $m=2$ is not in agreement with chemical evidence[14] for the rods and presumably also is not true for the cones.

THE EQUILIBRIUM EQUATIONS

Let us first consider the equations for the kinetics of the photochemical cycle

$$S\rightarrow E\rightarrow P+A\rightarrow S\cdots.$$

If the concentration of E is denoted by w, and that of P and A by x, the concentration of S will be $a-w-x$. The rates of these reactions will be

$$\frac{-d[S]}{dt}=\frac{-d(a-w-x)}{dt}=\frac{dw}{dt}=k_1I(a-w-x), \tag{3}$$

$$\frac{-d[E]}{dt}=\frac{-dw}{dt}=\frac{dx}{dt}=k_2w, \tag{4}$$

$$\frac{-d[P+A]}{dt}=\frac{-dx}{dt}=\frac{d(a-w-x)}{dt}=k_3x^2, \tag{5}$$

where k_1, k_2, and k_3 are velocity constants. At equilibrium the three rates are equal, and the equations become

$$k_1I(a-w-x)=k_2w=k_3x^2. \tag{6}$$

The solution for x in terms of I is

$$x=\frac{-k_1I+[k_1^2I^2+4(k_3+k_1k_3I/k_2)(k_1Ia)]^{\frac{1}{2}}}{2(k_3+k_1k_3I/k_2)}. \tag{7}$$

It is apparent from (6) that

$$x=(k_2w/k_3)^{\frac{1}{2}} \quad \text{and} \quad w=k_3x^2/k_2. \tag{8}$$

These equations yield the curves shown in Fig. 1. The curve for w is sigmoid and almost symmetrical. Those for x and $(a-w-x)$ are quite asymmetrical, and the curve for $(a-w-x)$ is almost identical with that for $(a-x)$ derived from the equilibrium equation of Hecht. A comparable set of equations was proposed by Byram.[4]

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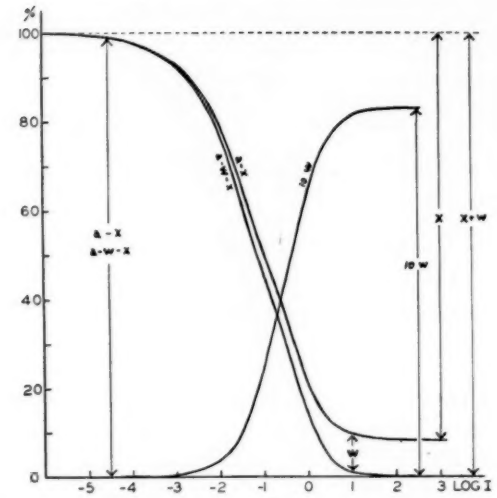

FIG. 1. Equilibrium percentages of $a-x$, $a-w-x$, and w as functions of log I, according to Eq. (6).

BRIGHTNESS DISCRIMINATION

The amount (Q) of photochemical product (E) formed by a flash of light is $k_1I'[S]t$.[19] On the basis of Eq. (3) this quantity is designated as $[k_1I'(a-w-x)]t$. In the case of experiments on brightness discrimination we may consider I' to be ΔI. The quantity of E formed by the additional light (which may be a brief flash or one of prolonged duration) is in addition to the equilibrium quantity already present as a result of the prevailing illumination. The equilibrium value of w is given by Eqs. (3)–(6). The amount of E which is effective during a flash is $Q+w$, a quantity which we shall designate as w'.

The rate of the physiological effect of E during a flash is equal to the product of velocity constant of the secondary reaction ($L\rightarrow T$) which we can designate as k_4, the concentration (l) of the substrate L, and the concentration (w') of E, i.e., is equal to k_4lw'. The total effect is the product of rate and time, i.e., $k_4lw't$. We may assume either that the rate or the total is constant, i.e., either k_4lw' or $k_4lw't=C$. Since the $L\rightarrow T$ reaction is either reversible or cyclic the assumption that the threshold corresponds to a certain rate seems permissible. Since t is maintained constant in experiments, the form of the $\Delta I/I-\log I$ curve will be the same on either assumption. Let us assume that $k_4lw'=C$. Then

$$C=k_4l[k_1\Delta I(a-w-x)t+w]^q, \qquad (9)$$

where q denotes the relationship between the rate of the secondary reaction and the response of the sense cell. From (9) we may write

$$\Delta I=\frac{(C/k_4l)^{1/q}-w}{k_1(a-w-x)t}. \qquad (10)$$

Dividing by I,

$$\frac{\Delta I}{I}=\frac{(C/k_4l)^{1/q}-w}{k_1I(a-w-x)t}. \qquad (11)$$

However in (6) $k_1I(a-w-x)=k_3x^2$. Therefore,

$$\frac{\Delta I}{I}=\frac{(C/k_4l)^{1/q}-w}{k_3x^2t}. \qquad (12)$$

Since, from (6), $w=k_3x^2/k_2$

$$\frac{\Delta I}{I}=\frac{(C/k_4l)^{1/q}}{k_3x^2t}-\frac{1}{k_2t}. \qquad (13)$$

Let us assume that l is the same at the beginning of each flash and can, therefore, be treated as a constant. In (12) if C is chosen so that the qth root of the parenthetical expression is only slightly higher than the maximum equilibrium value of w at extremely high intensities, then with increasing I the numerator decreases from approximately w_{max} to slightly more than zero. How closely this value approaches zero determines the curvature of the $\Delta I/I-\log I$ curve. The slope of the terminal low intensity portion, however, is determined mostly by x^n which is in turn determined by the equilibrium Eqs. (3)–(6).

In Fig. 2 are shown data for white and certain monochromatic light from Hecht, Peskin, and Patt.[16] The curve is drawn for Eq. (12) with $q=1$, $t=1$, and the parenthetical expression equal to 8.392. With $k_1=1000$, $k_2=2000$, $k_3=2$, the maximum value of w is about 8.392. Since the vertical position is determined by the ratio of the qth root of the parenthetical expression and k_3t, and the horizontal position by the relative values of k_1, k_2, and k_3, the curves may be freely shifted on either axis without change of shape. If necessary, the curvature of the middle section of the curve could be changed by changing C.

However, all of the data which were fitted by Eq. (2) can be fitted by the curves of Fig. 2.

The exponent q is introduced into these equations only for completeness. In the interpretation of experiments on critical flicker frequencies[19] it was necessary to assume different values of q for rods and cones. In the present equations, however, a change in q could be compensated for by a change in C without changing the shape of the curve.

On the basis of Eq. (1) it was necessary to assume $m=n=2$ for cone data and $m=2$, $n=1$ or 2 for rod data on brightness discrimination and visual acuity. In the new Eqs. (12) and (13) $m=1$, $n=2$ for all data. This seems to be in best agreement with available chemical evidence.

ACUITY AND OTHER VISUAL FUNCTIONS

Since the critical visual angle (α) varies as $\Delta I/I$[15] the form of the new equation applicable to visual acuity is

$$\alpha=C_1\frac{(C/k_4l)^{1/q}-w}{k_3x^2t}, \qquad (14)$$

where C_1 is a proportionality constant. This equation will fit all of the data which could be fitted by the equivalent form of Eq. (2), e.g., data of Shlaer[22] and Shlaer, Smith, and Chase.[23]

Equation (9) is one of general applicability to several visual functions. For instance, in experiments on dark adaptation where the intensity of a brief flash necessary to produce a certain response is determined as a function of time, the value of w is zero. Values of $a-x$ may be calculated from the experimentally determined values of $I(=\Delta I)$. These should give the course of the reaction $P+A\rightarrow S$. In experiments on instantaneous threshold[11–12] where the prevailing intensity is suddenly reduced to zero, the minimum value of $I(=\Delta I)$ which is immediately just barely detectable should follow Eqs. (11)–(13).

The new equations offer definite predictions concerning the result of changing the temperature (increases of k_2, k_3, and k_4) and the duration of the flash (t). At present, experimental evidences of the validity of the temperature predictions are not available, but the prediction for changing the value of t is in agreement with the brightness discrimination data for short flashes of Graham and Kemp[7] and with the acuity data of Graham and Cook[6] and Niven and Brown.[21]

SUMMARY

New equations for brightness discrimination and for visual acuity as functions of intensity have been derived on the basis of the photochemical theory. These equations can be fitted to all existing data and are not in conflict with the available chemical evidence. They may also be modified to fit data on dark adaptation and instantaneous threshold.

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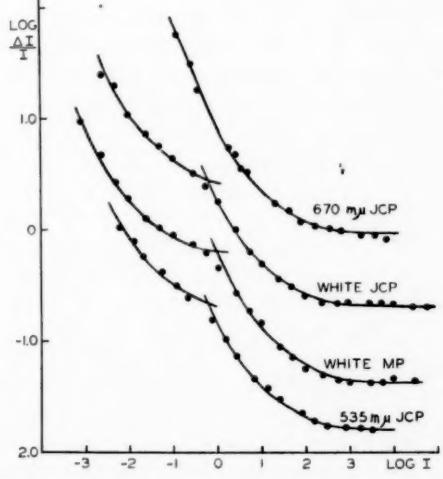

FIG. 2. Log $\Delta I/I$ as a function of log I, data for white and monochromatic light from Hecht, Peskin, and Patt (1938), curves calculated from Eq. (12).

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The Effect of Quality of Illumination on the Results of the Ishihara Test

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THIS is the third in a series of reports dealing with the Ishihara test for color-blindness. In the first of these[1] the fifth edition of the test was evaluated as a means of detecting and analyzing defective color vision. The data reported were collected during the course of a study of some 106 persons having defective color vision of varying types and amounts, including 74 who had definite color defects and 32 who had low, but normal, color vision. To study these persons a comprehensive battery of color tests[2] was used, some of which are well known in the field of color-blindness testing, some are less well known, and some were devised in this laboratory. The fifth edition of the Ishihara test, one of the best known of the shorter editions, was selected for inclusion in the battery of tests. The results obtained from the entire battery aided in the classification of the color defect as to type and extent and at the same time permitted us to evaluate each test of the battery both as a diagnostic or screening medium (the purpose of which is to screen out the color defectives from the color normals) and as a medium for classification (the purpose of which is to determine the type and extent of the defect).

Concerning the fifth edition of the Ishihara test it was concluded that, when properly administered, the test affords a good rough device for screening red-green defectives from color normal individuals if a performance score of 60 is taken as the critical score; that is, if 60 percent of the plates are read correctly. In the group tested no color defective scored more than 58 and no color normal less than 75. The Ishihara test is, however, only a gross test for defective red-green vision. It fails to classify the type of defective color vision (protanope, protanomalous, deuteranope, deuteranomalous, tritanope, tritanomalous[3]) and cannot be used to give a satisfactory evaluation of the extent of the defect, no matter how carefully administered.

In the second of these reports[4] a comparative evaluation was made of three representative editions of the Ishihara test (the fifth,[5] the seventh,[6] and the British reprint of the ninth[7]) and of the Ishihara plates included in the Ameri-

[1] L. H. Hardy, G. Rand, and M. C. Rittler, "Tests for the detection and analysis of color-blindness. I. The Ishihara test: An evaluation," J. Opt. Soc. Am. **35**, 268 (1945).
[2] See reference 1, 270–272.
[3] For a simple explanation of these terms, see reference 1, page 271; also L. H. Hardy and G. Rand, "Recent developments in color vision testing," Am. Acad. Ophth., Outline of Instructional Courses 1944. Graduate Lecture Continuous Course No. 6 and 7; L. H. Hardy, G. Rand, and M. C. Rittler, "Color vision and color vision testing," Arch. Ophth. (in press); D. B. Judd, "Facts of Color-Blindness," J. Opt. Soc. Am. **33**, 294 (1943).
[4] L. H. Hardy, G. Rand, and M. C. Rittler, "Tests for the detection and analysis of color-blindness. II. The Ishihara test: Comparison of editions," J. Opt. Soc. Am. **35**, 350 (1945).
[5] S. Ishihara, *Tests for Colour-Blindness* (Kanehara, Tokyo, 1925), fifth edition.
[6] S. Ishihara, *Tests for Colour-Blindness* (Kanehara, Tokyo, 1936), seventh edition.
[7] S. Ishihara, *Tests for Colour-Blindness* (reprinted, H. K. Lewis and Company, London, 1943), ninth edition.

can Optical Company's compilation of pseudo-isochromatic plates.[8] Twenty-two subjects to whom the entire battery of tests had been administered were available for that work. The general conclusion drawn was that on the whole the three editions of the Ishihara test yield the same type of results in spite of the changes that have been made in the number of plates and in the plates themselves, some of which are improved in one edition as compared with the others, while other plates are less effective. The seventh and ninth editions have been improved over the fifth edition insofar as detecting protanomalous and protanopic subjects is concerned; but the plates used for the classification of *type* of red-green defect are less successful for deuteranomalous and protanomalous subjects, particularly as reproduced in the seventh edition. None of the editions yields a differential score as to the *extent* of color defect. It seems probable from the data that the critical performance score of 60, marking the dividing line between low normal and defective color vision, holds for the seventh and ninth editions of the test. This critical score, it will be remembered, was established for the fifth edition on the larger group of subjects which included both color defective and low normal observers. The Ishihara plates appearing in the A.O. test were found, however, to be for the most part poor selections and poor reproductions of the original Japanese plates, and to have less value for screening deuteranomalous and deuteranopic subjects than the plates as reproduced by Ishihara.

The present report describes the effect produced on the results of the Ishihara test by changing the *quality of the illumination* under which the test is administered. A comparison was made between the two most commonly used illuminants: daylight, for which the tests were designed, and tungsten-filament light, which is far too frequently used because of carelessness or lack of proper facilities combined with ignorance of the important effect of quality of illumination on the results of all tests which employ test material seen by reflected light.

The critical importance of quality of illumination for all tests of this type and particularly for the polychromatic and pseudo-isochromatic plates which are being used for the detection of defective color vision, was stressed in the first report of this series,[1] and also independently by Hardy.[9] Because of their great convenience and apparent simplicity of administration, plates of this type have been widely employed in military and industrial fields for many years. We expressed the belief that part of the dissatisfaction that has been reported with regard to the findings of these tests has been due to disregard of the critical importance of the illumination under which the tests were administered. As already indicated, tests of this type were designed to be administered under daylight, and instructions to that effect are given. Too often, however, they have been carelessly administered under artificial light, usually tungsten-filament light. The composition of this light, as compared with average daylight, is relatively strong in the long wave-length portion of the spectrum and weak in the short wave-length portion. Consequently when tungsten light is employed instead of daylight, there will be enhancement of red and yellow pigments, dulling of green and blue pigments, and a shift in hue of the mixed colors toward the long wave-lengths.

To determine the effect of this incorrect administration, color defective subjects were tested with different editions of the Ishihara test utilizing the same amounts (25 ft.-c) of different illuminants. The responses were compared when the illuminant was changed from artificial daylight to tungsten-filament light. The daylight illumination was obtained from the nearest practical equivalent to I.C.I. Illuminant *C*, that provided by a Macbeth daylight lamp (tungsten lamp plus filter of Corning Daylite glass) designed to operate at approximately 6750°K. The tungsten-filament illumination was provided by a 75-watt Mazda gas-filled lamp operating at rated voltage, color temperature approximately 2848°K. This illumination is equivalent to I.C.I. Illuminant *A*. For convenience, these illuminations will be called "Daylight" and "Tungsten" throughout the present report.

[8] American Optical Company, *Pseudo-Isochromatic Plates for Testing Color Perception* (Beck Engraving Company, Philadelphia and New York, 1940).

[9] L. H. Hardy, "Standard illuminants in relation to color-testing procedures," Arch. Opthal. **34**, 278 (1945).

TABLE I. Effect of quality of illumination on responses to the individual plates. Percentage of each type of color defective subjects passing each plate of the Ishihara test, editions 5, 7, and 9 (British reprint), and the Ishihara plates in the American Optical Company's test, when the test is administered under daylight and under tungsten light. Each plate is designated by series number and plate number as it appears in the edition under consideration.

Type of color defective subject	Number of cases	Quality of illumination	Percentage of each type passing individual plates
			Series no. 1
			5th edition (12 plates)
			Plate no. 2 3
Anomalous trichromat			
Deuteranomalous	7	Daylight	57 72
		Tungsten	*86 72*
Protanomalous	5	Daylight	80 100
		Tungsten	*80 60*
Dichromat			
Deuteranope	5	Daylight	0 0
		Tungsten	*0 0*
Protanope	5	Daylight	60 60
		Tungsten	*60 20*
All color defectives	22	Daylight	50 59
		Tungsten	*59 41*

Type of color defective subject	Number of cases	Quality of illumination	Percentage of each type passing individual plates
			Series no. 1
			7th edition (24 plates)
			Plate no. 2 3 4 5
Anomalous trichromat			
Deuteranomalous	7	Daylight	43 14 0 14
		Tungsten	*72 29 0 29*
Protanomalous	5	Daylight	0 20 0 0
		Tungsten	*0 20 0 0*
Dichromat			
Deuteranope	5	Daylight	0 0 20 0
		Tungsten	*0 0 0 0*
Protanope	5	Daylight	0 20 0 0
		Tungsten	*0 0 0 0*
All color defectives	22	Daylight	14 14 5 5
		Tungsten	*23 14 0 9*

Type of color defective subject	Number of cases	Quality of illumination	Percentage of each type passing individual plates
			Series no. 1
			9th edition, British Reprint (24 plates)
			Plate no. 2 3 4 5
Anomalous trichromat			
Deuteranomalous	7	Daylight	43 43 0 14
		Tungsten	*29 43 14 29*
Protanomalous	5	Daylight	20 20 0 0
		Tungsten	*0 20 20 0*
Dichromat			
Deuteranope	5	Daylight	0 0 0 0
		Tungsten	*20 20 0 0*
Protanope	5	Daylight	0 20 0 0
		Tungsten	*20 20 0 0*
All color defectives	22	Daylight	18 23 0 5
		Tungsten	*18 27 9 9*

Type of color defective subject	Number of cases	Quality of illumination	Percentage of each type passing individual plates
			Series no. 1
			Ishihara Plates, A.O. Test (10 plates)
			Plate no. 5 6 11 12
Anomalous trichromat			
Deuteranomalous	7	Daylight	86 43 0 43
		Tungsten	*100 72 29 72*
Protanomalous	5	Daylight	40 0 0 0
		Tungsten	*60 0 0 20*
Dichromat			
Deuteranope	5	Daylight	20 20 0 20
		Tungsten	*60 40 0 40*
Protanope	5	Daylight	40 0 0 0
		Tungsten	*40 0 0 0*
All color defectives	22	Daylight	50 18 0 18
		Tungsten	*68 32 9 36*

Type of color defective subject	Number of cases	Quality of illumination	Percentage of each type passing individual plates
			Series no. 1
			Ishihara Plates, A.O. Test (10 plates)
			Plate no. 19 20 41 42
Anomalous trichromat			
Deuteranomalous	7	Daylight	72 0 72 57
		Tungsten	*86 86 100 100*
Protanomalous	5	Daylight	0 0 0 0
		Tungsten	*20 0 40 40*
Dichromat			
Deuteranope	5	Daylight	0 0 40 40
		Tungsten	*100 100 100 100*
Protanope	5	Daylight	0 0 0 0
		Tungsten	*0 0 60 40*
All color defectives	22	Daylight	23 0 32 27
		Tungsten	*55 50 77 73*

Type of color defective subject	Number of cases	Quality of illumination	Percentage of each type passing individual plates
			Series no. 1
			Ishihara Plates, A.O. Test (10 plates)
			Plate no. 33 34
Anomalous trichromat			
Deuteranomalous	7	Daylight	57 57
		Tungsten	*29 43*
Protanomalous	5	Daylight	20 20
		Tungsten	*20 40*
Dichromat			
Deuteranope	5	Daylight	20 20
		Tungsten	*60 60*
Protanope	5	Daylight	0 0
		Tungsten	*20 20*
All color defectives	22	Daylight	27 27
		Tungsten	*32 41*

The effect of this change in illumination on the results of the test was determined for the fifth, seventh, and ninth (British reprint) editions of the Ishihara test and for the Ishihara plates included in the American Optical Company's compilation of pseudo-isochromatic plates. Only the plates intended for testing color-blind literates were used, of which there are 12 in the fifth edition, 24 in the seventh and ninth editions, and 10 in the A.O. test. These plates consist of six series. *Series 1 and 2* are of the "transformation pattern" type, in which the color defective subject sees a different pattern from that which the color normal subject sees; *Series 3 and 4* are of the "vanishing digit" type, in which the pattern is not seen at all by the color defective subject; *Series 5* contains a "hidden digit" which is supposedly seen only by a color defective subject; and *Series 6* is intended for the differential classification of type of red-green defect. A fuller description of the six series of plates is given in the first report of this series.[1] In the fifth edition there are 2 plates of each series; in the seventh and ninth editions, 4 plates of each series; and in the A.O. test, 4 plates of *Series 1*, 4 of *Series 2*, and 2 of *Series 5*. Differences found in the effect of change of illumination from edition to edition are, of course, due to differences in the pigments used, which in some instances are quite marked.

Twenty-two subjects to whom the entire battery of tests[2] had been administered were available for the present work. They included 12 anomalous trichromats, of whom 7 were deuteranomalous and 5 were protanomalous; and 10 dichromats, of whom 5 were deuteranopes and 5 were protanopes. Differences in the effect of change of illumination found between the different types of color defectives are due both to the extent or degree of the defect and to the characteristic reaction of each type to the color combinations used in the various plates of the Ishihara test.

Throughout the work the testing distance was about 30 inches. Care was taken to avoid any specular reflection from the test plates. To this end, the angle of illumination was approximately 45° and the angle of view 90°.

The general conclusion to be drawn from the study is that performance scores on the Ishihara test tend to be higher for red-green defectives (i.e., a greater number of correct responses are given) when the illuminant is tungsten light than when it is daylight, most significantly in the cases of deuteranomalous and deuteranopic subjects. The effect is so marked that a substantial percentage of deuteranomalous subjects (42 percent) score within normal limits when tungsten light is employed. Further, the Ishihara test is a poorer medium for the differential classification of type of red-green defect when the illuminant is tungsten light. Fewer subjects are correctly classified by the test, more are not classified at all, and a greater number of protanomalous and protanopic subjects are incorrectly classified when the illumination is shifted from daylight to tungsten.

In detail, the results are summarized in Tables I–III. They will be discussed under 3 headings: 1. Effect of quality of illumination on responses to the individual plates; 2. Effect of quality of illumination on performance scores on the entire test; and 3. Effect of quality of illumination on the differential classification of deuteranopia and protanopia, deuteranomaly and protanomaly. In each table the results for daylight illumination are given in roman type; those for tungsten illumination in italics.

1. EFFECT OF QUALITY OF ILLUMINATION ON RESPONSES TO THE INDIVIDUAL PLATES

Table I shows for each type of color defect and for each illuminant the percentage of cases which pass the individual plates of each edition, the plates being designated by both series number and plate number as they appear in the edition under consideration. The first four horizontal rows of figures in roman type under each edition show these percentages for each type of color defect when the test is administered under daylight; the first four rows in italics under each edition show these percentages when the test is administered under tungsten light. These data demonstrate the value of the individual plates for detecting each type of color defective subject when the test is administered under the illuminant indicated. The two bottom rows under each edition show the percentages for the color defectives as a group and demonstrate the value

of the individual plates for the simple detection of defective color vision.

The following statements summarize the more marked changes produced by administering the tests under tungsten instead of daylight illumination:

1. In general a higher percentage of those having defective color vision pass the individual plates of the Ishihara test when tungsten illumination instead of daylight is used. This result is most marked for deuteranomalous subjects and is particularly evident with the plates as reproduced in the fifth edition. It is next most significant for deuteranopic subjects, especially with the Ishihara plates as reproduced in the A.O. selection. In other words, fewer red-green defectives, particularly those of the deuteranomalous and deuteranopic types, are detected by the plates of the Ishihara test when it is incorrectly administered under tungsten illumination.

2. Without entering into a detailed analysis of the changes produced in the appearance of each plate as the result of the change in illuminant from daylight to tungsten light, the following comments are probably justified with regard to the factors which produce a higher percentage of normal responses among red-green defectives when the test is administered under tungsten illumination:

Series 1 and 2 (transformation pattern): In the plates of *Series 1* one or two digits, composed in general of blue-red and yellow-red disks, are seen by the color normal on a background composed of blue-green and yellow-green disks. To the red-green defective, to whom the yellow-red portion of the digit and the yellow-green background have little if any differentiation, only the blue-red and blue-green disks stand out from the background, forming for him a bluish pattern on a yellowish background. The pattern is thus transformed for the color defective into a different one from that seen by the color normal; e.g., from an eight to a three. In the plates of *Series 2* approximately the same combination of colors is used but with background and pattern reversed; that is, blue-green and yellow-green disks form the digits on a background of blue-red and yellow-red disks. Again the pattern seen by the red-green defective subject is formed by those disks which have a blue component. Since the effect of tungsten light is to enhance red and yellow pigments, to dull green and blue pigments, and to shift the hue of mixed colors toward the longer wave-lengths, the digits seen by the color normal are emphasized and those seen by the red-green defective are dulled by the use of this illuminant. The result is that fewer red-green defectives give the expected response when the illumination is changed from daylight to tungsten light. Instead they more frequently give the normal response and obtain a passing score, as is shown in Table I; or else they often see no digit at all and fail the test—a change in response which is not shown in Table I but is revealed by an analysis of individual responses.

The plates of *Series 2* are most affected by the change in illumination as reproduced in the ninth edition (British reprint) of the Ishihara test and in the A.O. selection of Ishihara plates—some of the plates of *Series 2* of these editions being passed by all the deuteranomalous and deuteranopic subjects.

Series 3 and 4 (vanishing digit): In the plates of these series the contrast between digit and background is obviously intensified under tungsten light as compared with daylight. The result is that fewer color defectives fail to read the digit; thus more succeed in passing the plates.

Series 5 (hidden digit): In the plates of this series the digit which is supposedly seen only by color defective subjects is bluish in color. Under tungsten illumination this coloration is less pronounced than under daylight. The result is that fewer color defectives see this bluish "hidden digit," and thus again more of them give the normal response (no digit seen) and pass the plates. It may be mentioned that this hidden digit is seen under daylight illumination by many having normal color vision who, by this reaction, fail to pass the plate. It is rarely visible to the color normal, however, when viewed by him under tungsten light.

In all editions of the Ishihara test studied, the responses given by the deuteranomalous and deuteranopic subjects to the plates of this series are significantly affected by the change in illumination. Most deuteranopes, and with some plates *all* the deuteranopes, failed to see the pattern under tungsten illumination, thus passing the

TABLE II. Effect of quality of illumination on performance scores on the Ishihara test, showing for each type of color defective subjects (a) the average performance score achieved by each group; (b) the median or middle score for each group; and (c) the range of scores from lowest to highest achieved by the individuals within each group, when the test was administered under daylight and under tungsten light. *Performance score is percentage of the plates to which correct responses were given.*

Type of color defective subject	Number of cases	Quality of illumination	Average score	Median score	Range of scores
		5th edition			
Anomalous trichromat					
Deuteranomalous	7	Daylight	20	17	0–58
		Tungslen	*59*	*58*	*25–92*
Protanomalous	5	Daylight	18	17	8–33
		Tungslen	*28*	*33*	*8–42*
Dichromat					
Deuteranope	5	Daylight	0	0	0
		Tungslen	*15*	*8*	*8–25*
Protanope	5	Daylight	13	17	8–17
		Tungslen	*15*	*17*	*0–25*
All color defectives	22	Daylight	14	17	0–58
		Tungslen	*32*	*25*	*0–92*
		7th edition			
Anomalous trichromat					
Deuteranomalous	7	Daylight	18	17	0–33
		Tungslen	*44*	*42*	*21–67*
Protanomalous	5	Daylight	8	12	0–17
		Tungslen	*7*	*4*	*4–17*
Dichromat					
Deuteranope	5	Daylight	2	4	0– 4
		Tungslen	*26*	*25*	*13–37*
Protanope	5	Daylight	4	0	0–17
		Tungslen	*7*	*4*	*4–17*
All color defectives	22	Daylight	9	6	0–33
		Tungslen	*23*	*19*	*4–67*
		9th edition (British Reprint)			
Anomalous trichromat					
Deuteranomalous	7	Daylight	19	21	0–42
		Tungslen	*30*	*25*	*21–54*
Protanomalous	5	Daylight	6	4	0–12
		Tungslen	*11*	*12*	*0–21*
Dichromat					
Deuteranope	5	Daylight	2	0	0– 8
		Tungslen	*27*	*29*	*21–29*
Protanope	5	Daylight	5	4	0–17
		Tungslen	*9*	*4*	*4–17*
All color defectives	22	Daylight	9	4	0–42
		Tungslen	*20*	*21*	*0–54*
		Ishihara Plates, A.O. Test			
Anomalous trichromat					
Deuteranomalous	7	Daylight	50	50	20–70
		Tungslen	*71*	*80*	*50–100*
Protanomalous	5	Daylight	8	0	0–30
		Tungslen	*24*	*20*	*10–40*
Dichromat					
Deuteranope	5	Daylight	18	10	0–50
		Tungslen	*66*	*60*	*40–90*
Protanope	5	Daylight	4	0	0–10
		Tungslen	*18*	*20*	*0–30*
All color defectives	22	Daylight	23	10	0–70
		Tungslen	*47*	*45*	*0–100*

particular plate. The responses given by the protanomalous subjects and in the majority of cases by the protanopic subjects, however, were not affected by the change of illumination. That is, they saw the hidden digit under both daylight and tungsten light, thus failing the plate. The reason for this may perhaps be found in the tendency of protanomalous and protanopic subjects to see the hidden digit, which is bluish in hue, as lighter than the background, which is in general composed of red, orange, and yellow disks. These background colors, it will be remembered, appear darker to protanomalous and protanopic subjects than they do to deuteranomalous, deuteranopic, and normal persons.

Series 6 (diagnostic plates): These plates carry two digits (e.g., 26), one in red and one in red-purple disks on a background of gray disks of different brightnesses. Protanopes, who have a neutral band in the red of the normal color system, just too purplish to be a spectral red, are supposed to see only the red-purple digit (the 6); and deuteranopes, who have a neutral band in the red-purple of the normal color system well outside the limits of the visible spectrum, are supposed to see only the red digit (the 2). The effect of the incorrect use of tungsten illumination on the plates of this series is to intensify the red component of both digits and to dull the blue component of the red-purple digit. The result is that more color defective subjects see both digits correctly against their gray background, and thus more succeed in passing some at least of the plates of this series. This result occurred among deuteranomalous, protanomalous, and protanopic subjects. It did not occur with any of our deuteranopes.

The effect of the change in quality of the illumination from daylight to tungsten light on the diagnostic possibilities of the plates of *Series 6* is discussed in a later section.

2. EFFECT OF QUALITY OF ILLUMINATION ON PERFORMANCE SCORES ON THE ENTIRE TEST

Table II shows the performance scores obtained on the entire test for each of the editions of the Ishihara test under consideration and for the 10 Ishihara plates in the A.O. test when administered under daylight and under tungsten light. In this table are given for each type of color defect (a) the average score achieved by the group; (b) the median or middle score for

each group; and (c) the range of scores from lowest to highest achieved by the individuals within each group. A score of 100 indicates that correct responses were given to all the plates of the test; a score of 0 that no correct responses were given; and scores between 0 and 100 indicate the percentage of plates to which correct responses were given. As in Table I the first four rows of figures in roman type under each edition give the data for each type of color defect when the test is administered under daylight; the first four rows in italics under each edition give the data when the test is administered under tungsten light. These data thus demonstrate the effect of changing the quality of illumination from daylight to tungsten on the scores achieved on the entire test for each of the types of color defectives used as subjects. The two bottom rows of figures under each edition show this effect for the color defectives as a group and demonstrate the significance of performance scores on the entire test for the simple detection of the presence of defective color vision when the test is administered under the two types of illumination.

Table II demonstrates:

1. With the single exception of the protanomalous subjects tested with the seventh edition of the Ishihara test, the average performance scores are always higher when the test is administered under tungsten light than when it is administered under daylight. The improvement in score is most marked for deuteranomalous subjects, particularly with the fifth edition of the test, and for deuteranopes, particularly with the A.O. Ishihara plates. The change in average score with protanomalous and protanopic subjects is of little consequence for all editions, except perhaps the A.O. Ishihara plates.

2. With the exception of the protanomalous subjects when tested with the seventh edition and of the protanopes with the ninth edition, the median performance scores also are higher when the test is administered under tungsten light. Again the increase in score is most marked with deuteranomalous and deuteranopic subjects.

3. In conformity with the higher performance scores attained by most red-green defectives under tungsten light, the range of scores from lowest to highest is in general shifted upward and in many instances is widened when this illuminant is used. This is shown in the last column of Table II. As is seen in this column, the upper value of this range for deuteranomalous subjects tested with the fifth edition of the Ishihara test is shifted above the critical score of 60, the score that was established in an earlier report[1] as the dividing line between normal performance and the performance of color defective subjects when the test is properly administered under daylight.

An analysis of individual cases among this group of red-green defective subjects reveals that, of the seven tested, three or 42 percent would not have been screened from the normal under tungsten illuminant. The scores for these three cases jumped, respectively, from 17 to *92*; from 58 to *83*; and from 25 to *75* when the illumination was shifted from daylight to tungsten light. In other words, the criterion of 60 percent performance on the Ishihara test, fifth edition, cannot be used when tungsten light is employed.

In order to show that the Ishihara test is not unique among pseudo-isochromatic tests in being influenced by the quality of illumination under which the test is administered, the following brief statement of results obtained with the complete A.O. test[8] may be given. As was the case with the Ishihara test, only the plates designed for literate subjects were used. For the same 22 color defective subjects whose comparative scores on the Ishihara test are given in this report, the following average scores and ranges of scores for the entire A.O. test were obtained when the illuminant was changed from daylight to tungsten light:

Deuteranomalous—average score increased from 37 to *52;* range of scores from 28–55 to *30–70;*

Protanomalous—average score increased from 28 to *33;* range of scores changed from 20–35 to *25–43;*

Deuteranope—average score increased from 17 to *35;* range of scores changed from 15–23 to *25–48;*

Protanope—average score increased from 16 to *18;* range of scores changed from 5–20 to *13–25.*

The improvement in performance scores on the A.O. test, when the illuminant is shifted from daylight to tungsten, almost precisely parallels that noted on the Ishihara test. That is, the

TABLE III. Effect of quality of illumination on the differential classification of deuteranopia and protanopia, deuteranomaly and protanomaly. Analysis of the value of the plates of *Series 6* of the 5th, 7th, and 9th (British reprint) editions of the Ishihara test as a means of differentiating between types of red-green defect, when the test is administered under daylight and under tungsten light.

Type of color defective subject	Number of cases	Quality of illumination	Correctly classified by at least 50% of the plates	Incorrectly classified by at least 50% of the plates	Not classified by at least 50% of the plates
		5th edition	%	%	%
Anomalous trichromat					
Deuteranomalous	7	Daylight	86	0	14
		Tungsten	*57*	*0*	*43*
Protanomalous	5	Daylight	80	0	20
		Tungsten	*0*	*80*	*20*
Dichromat					
Deuteranope	5	Daylight	100	0	0
		Tungsten	*100*	*0*	*0*
Protanope	5	Daylight	60	0	40
		Tungsten	*40*	*20*	*40*
		7th edition			
Anomalous trichromat					
Deuteranomalous	7	Daylight	57	0	43
		Tungsten	*43*	*0*	*57*
Protanomalous	5	Daylight	40	20	40
		Tungsten	*0*	*60*	*40*
Dichromat					
Deuteranope	5	Daylight	100	0	0
		Tungsten	*100*	*0*	*0*
Protanope	5	Daylight	60	0	40
		Tungsten	*0*	*20*	*80*
		9th edition (British Reprint)			
Anomalous trichromat					
Deuteranomalous	7	Daylight	86	0	14
		Tungsten	*86*	*0*	*14*
Protanomalous	5	Daylight	60	0	40
		Tungsten	*0*	*60*	*40*
Dichromat					
Deuteranope	5	Daylight	100	0	0
		Tungsten	*100*	*0*	*0*
Protanope	5	Daylight	60	0	40
		Tungsten	*40*	*40*	*20*

improvement was most marked for deuteranomalous and deuteranopic subjects.

3. EFFECT OF QUALITY OF ILLUMINATION ON THE DIFFERENTIAL CLASSIFICATION OF DEUTERANOPIA AND PROTANOPIA, DEUTERANOMALY AND PROTANOMALY

As was stated previously, the plates of *Series 6* of the Ishihara test were designed as a means of classifying *type of defect* among red-green defective subjects. On each plate of this series, there are two digits, the left one being composed of red disks lighter in brightness than the background gray disks and the right one of red-purple disks of approximately the same brightness as the background disks. According to Ishihara, the deuteranope type of red-green defective misses the red-purple digit and is able to read only the lighter red digit, since this type has a neutral band in the red-purple of the normal color system and has luminosity values that approximate those of normal subjects. The protanope type of red-green defective, on the other hand, misses the lighter red digit and is able to read only the red-purple digit, since this type has a neutral band nearer the red of the spectrum and perceives red as darker in brightness than do normal and deuteranopic subjects. The effectiveness of these plates for this classification as reproduced in the fifth, seventh, and ninth editions of the test was analyzed in previous reports[1, 4] when the test was administered under daylight, as directed by Ishihara. Table III gives the results for these editions of the test when the illumination was changed from daylight to tungsten light. This change in illumination causes a noticeable shift in the hue of the red-purple digit towards red, and a shift in the hue of the red digit towards yellow-red.

In scoring the responses to these plates, we used the criterion previously discussed.[1] That is, a case was called "classified" as to type of defect when one digit was read correctly, and the other not seen at all or read incorrectly. When both digits were read correctly, when neither was read correctly, or when one was read incorrectly and the other not seen at all, the case was called "not classified." In short, a "hit and miss" are necessary in order to utilize any plate of *Series 6* as a test for classification of type of red-green defect. It seems reasonable to require a "hit and miss" on at least 50 percent of the plates used for this purpose.

Table III indicates:

1. The percentage of deuteranomalous subjects correctly classified was decreased by the use of tungsten illumination. For the fifth edition the decrease was from 86 to *57* percent; for the seventh edition, from 57 to *43* percent. For the ninth edition there was no change. Since with this type of red-green defectives no incorrect classifications were made, the percentages for which no classification could be made were pro-

portionately increased. This result was due to the ability of more of the deuteranomalous subjects to read *both the red and the red-purple digit* under tungsten illuminant, when only the *red digit* was seen under daylight. Apparently the hue of the red-purple digit was shifted by the yellower illumination into a color region to which these subjects were more sensitive.

2. The percentage of deuteranopes correctly classified was not changed by the use of tungsten illumination. All the deuteranopes were correctly classified by at least 50 percent of the plates of *Series 6* under both illuminations.

3. In the classification of protanomalous and protanopic subjects, the deleterious effect of the yellower illumination is most strikingly demonstrated. The result was a decrease in the percentage of correct classifications and, more significantly, an increase in the occurrence of incorrect classifications. In the latter case the red digit, which was not seen by some of the protanomalous and protanopic subjects under daylight illumination, was seen under tungsten light because under this light its hue was apparently shifted towards yellow by a sufficient amount to bring it into a color region to which these subjects were *more* sensitive; while the red-purple digit, usually seen by them under daylight illumination, was not seen under tungsten light because its hue was apparently shifted towards red by a sufficient amount to bring it into the color region to which these subjects were *less* sensitive. Thus an incorrect classification of deuteranopia instead of the correct one of protanopia was more frequently obtained under tungsten light than under daylight. This result was particularly marked for the protanomalous subjects when tested with all three editions of the Ishihara test.

CONCLUSIONS

In spite of the small number of subjects used in this comparative study of the effect of quality of illumination on the results of the Ishihara test taken as typical of polychromatic and pseudo-isochromatic tests, the following conclusions seem to be justified when the test is incorrectly administered under tungsten light, instead of under daylight as is directed by the author of the test:

1. When tungsten light is used as the illuminant, the performance scores attained by all the deuteranomalous and deuteranopic subjects tested are higher than when daylight is used. The responses of protanopic and protanomalous subjects are little affected by this change in quality of illumination.

2. A substantial number of deuteranomalous subjects are sufficiently aided in giving normal responses by the incorrect use of tungsten light to take them out of the class of color defectives when judged by performance score; that is, their scores fall within the normal limits established for daylight illumination.

3. There is a decrease in the number of cases which are correctly classified for type of red-green defect under tungsten light as compared with daylight, and an increase in the number of protanomalous and protanopic cases which are incorrectly classified.

We wish to stress once more the critical importance of strict observance of correct conditions of illumination during the administration of polychromatic tests which employ test material seen by reflected light.

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The Aging of Glass Surfaces

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(Received November 9, 1945)

THAT glass surfaces are subject to deterioriation is attested by the appearance of ancient Roman and other glasses. Their surfaces are often rough and coated with iridescent films of rare beauty. The question has arisen as to the degree of deterioration or "aging" of modern optical glasses. This work is the outgrowth of some earlier observations made with an improved instrument[1] for determining refractive indices by means of the Brewsterian angle. It was found that the (Brewsterian) refractive indices (n_B) of glass surfaces at least fifty years old were absurdly low. However, as soon as these surfaces were reground and polished, the normal refractive index was obtained.

The subject-matter following is to be discussed under the headings:

1. Aging—normal and accelerated.
2. Identification of surface films.
3. Effect of surface films on the performance of optical instruments.
4. Optical properties of silica skeleton films on glass.

While the reader is referred to the previously cited article for a description of the Brewsterian angle apparatus, it is desired to make some additional comments. When photometric balance has been established it is found that, upon rotating the polarizer through the "horizontal" position, a black shadow in the form of an erect or inverted V sweeps vertically across the field. When the glass surface to be tested is fresh, the V is very black, but when a surface film is present, the V is distinctly less black. If, in the latter case, the green filter be removed, it is found that the two halves of the field show complementary colors—usually yellow and blue. From elementary considerations it can be shown that surface films of finite thickness must transform incident plane-polarized light into elliptically polarized light. This is one of the most sensitive tests known to the writer for detecting the existence of surface films on glass. In spite of the presence of such films, it is still possible to establish a photometric match by using the green filter. It is, however, recognized that no importance is attached to the numerical values of n_B—the sole purpose of the measurements being to obtain information as to changes in thickness of the surface film.

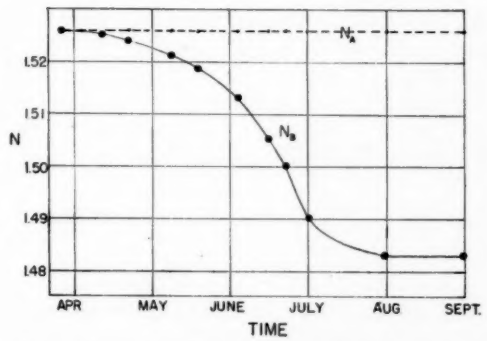

FIG. 1. Showing the change in the Brewsterian refractive index (N_B) with time (normal aging). Jamin interferometer plate $n = 1.52$.

I. AGING—NORMAL AND ACCELERATED

According to our concept, glass is looked upon as having aged normally if it is exposed, in the average physical laboratory, to ambient air at atmospheric pressure and at room temperature. Attention to the subject of aging was first drawn by a fifty-year-old Jamin interferometer plate whose refractive index (n_B) was 1.48 on the aged side and 1.52 on the fresh side. More recently the fresh side was allowed to age normally over a period of months. The changes of n_B with time are shown in Fig. 1. It will be noted that, whereas the values of n_B show marked changes with time, the values of n_A (obtained with the Abbe refractometer) remain constant.

So as to determine, if possible, the cause of this aging, a fresh surface was formed on the same Jamin plate, which was then cut in two: the one half being placed in a desiccator (with $CaCl_2$), and the other half in a humidor with saturated water vapor—all at room temperature.

[1] A. H. Pfund, J. Opt. Soc. Am. **31**, 679 (1941).

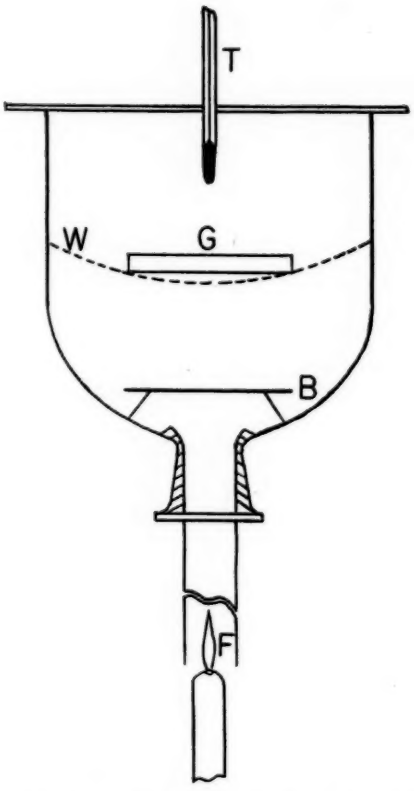

FIG. 2. Apparatus for accelerated aging of glass surfaces. F is a small gas flame; B is a baffle; G is the glass plate to be aged.

In four months no measurable change in n_B could be detected in either case. Now, these conditions differed from those shown in Fig. 1 only that, in the latter case, the air was free to circulate and to bring on a continuous supply of contaminating impurities. In the average physical laboratory the principal impurities are the products of combustion of a bunsen burner. On this basis the apparatus for accelerated aging, shown in Fig. 2, was constructed. This apparatus is to be termed the "lethal chamber." It consists of a small pointed (blue) gas flame F one inch long mounted at the lower end of a Pyrex tube about 1 cm in diameter and about 30 cm long. The upper end is cemented into the neck of a bottle with plaster of paris. This bottle, which had been cracked in two with a hot wire, is provided with a baffle B and a wire gauze W on which the (preheated) glass samples G are laid. The temperature, as read off the thermometer T, lies between 65° and 75°C. An asbestos board is laid over the open end of the bottle around which several layers of paper or cloth are wrapped.

To gain information as to the effectiveness of this mode of aging, a fresh surface was again formed on the Jamin plate, which was placed in the lethal chamber. The results were much the same as those shown in Fig. 1 with the difference that the drop in n_B from 1.52 to 1.48 required only nine hours instead of four months. This means that approximately a day in the lethal chamber brings about as much aging as does one year of normal exposure.

In the lethal chamber, readily attacked glasses develop a whitish film which can be removed easily by rubbing with a moist cloth. However, there remains a second, hard film which often shows interference colors. That the normally aged surface of the Jamin plate showed no such colors is due to the small difference in refractive index between film and underlying glass. That this film is thick enough to show interference colors was proved by mounting the aged surface in contact with a thin brass plate having a central hole. The plate was trimmed so that it fit snugly (diametrally) into a glass tube (4.0-cm diameter) containing a liquid whose refractive index was adjusted to 1.52. Since now the aged film lies between two media having the same refractive index, conditions for the development of interference colors are favorable. As a matter of fact, brilliant interference colors were observed.

Of the various glasses tested in the lethal chamber, those with high silica content were most resistant. Pyrex was best, crown glasses next, and (lead) flint glasses much the poorest. The performance of ordinary quarter-inch plate glass was remarkable. Exposure for several days in the lethal chamber, in moderately strong acid and alkali, failed to produce any detectable aging.

II. IDENTIFICATION OF SURFACE FILMS

Vasicek[2] has shown that the film produced by acid on the surface of a (lead) flint glass has a refractive index of 1.46—which is identical with that of fused silica. This means that the surface

[2] A. Vasicek, Phys. Rev. **57**, 925 (1940).

film on aged glass is due to the removal of more readily attacked constituents of the glass and the remaining of an inert film of pure, amorphous silica. As an independent check on the above conclusions, the problem was attacked by studying infra-red reflection from fresh and aged glass surfaces. For such work a small rocksalt spectrometer with vacuum thermopile and galvanometer were used. As has long been known, glasses have reflection maxima near 9μ—the form of the reflection curve for a fresh surface being characteristic of the complex silicates, oxides, etc. present in a particular glass. If, in consequence of aging, a film of silica overlies the surface, then an infra-red reflection-curve showing some of the characteristics of fused silica ought to result. That this is actually the case is shown in Figs. 3 and 4. The parallelism between the reflection curve for fused silica (A) and that of the aged glass side, B, as shown in Fig. 3, is particularly striking.

It will be noted that, regardless of the form of the reflection curve for the fresh surface, the aged surface invariably reveals a reflection maximum near 9.25μ which coincides with the principal reflection maximum of fused silica. These observations, coupled with those of Vasicek, make it appear highly probable that the surface film on aged glass is fused silica.

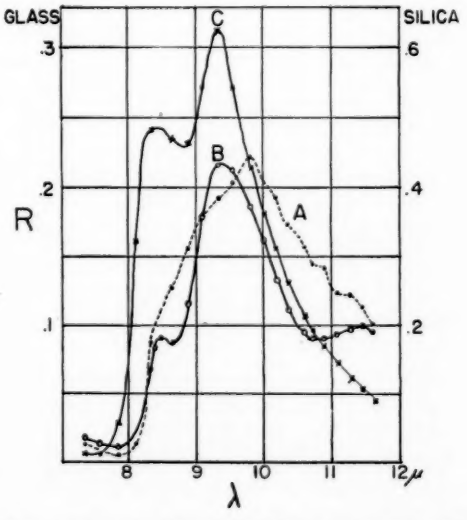

FIG. 3. A is the infra-red reflection curve for a fresh glass surface ($n=1.52$), B is the infra-red reflection curve for the (normally) aged surface, C is the infra-red reflection curve for fused silica.

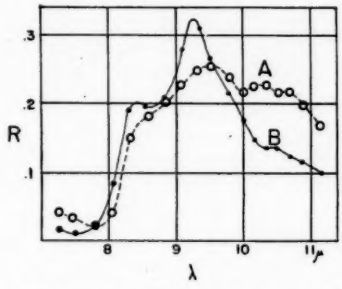

FIG. 4. A is the reflection curve for a fresh glass surface, B is the reflection curve for a normally aged surface. The glass, which was taken from an old New England side board, is known to be over 100 years old.

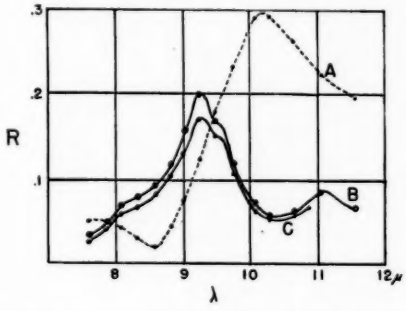

FIG. 5. A is the reflection curve for the fresh surface ($n=1.77$), B is the reflection curve for a normally aged surface (50 years), C is the reflection curve after a fresh surface had been left in the lethal chamber for 24 hours.

It might be questioned whether accelerated aging brings about the same chemical changes in a glass surface as does normal aging. Insofar as infra-red reflection is concerned, Fig. 5 is offered in evidence. These curves apply to a (lead) flint glass one of whose sides had aged normally for at least fifty years. The fresh side was then allowed to age in the lethal chamber for 24 hours. The similarity between curves B and C is too striking to be without significance. The conclusion that the two modes of aging produce the same type of surface film is supported by the following observations:

1. The infra-red reflection curves are essentially identical.
2. For films of the same thickness n_B is the same.
3. Prolonged exposure of normally aged films in the lethal chamber produces no change in the infra-red reflection curves or in n_B.

III. EFFECT OF SURFACE FILMS ON THE PERFORMANCE OF OPTICAL INSTRUMENTS

Surface films, unless they increase light losses by reflection, are virtually without influence on the performance of optical instruments. There is, however, one instrument, the Abbe refractometer, which is affected adversely. When the prism with the polished hypothenuse acquires a surface film of such thickness that interference colors are shown, the line of demarcation between the halves of the visual field first becomes fuzzy and, ultimately, disappears completely when liquids of refractive index 1.6 or greater are used. This defect is immediately remedied by repolishing this hypothenuse surface.

Another approach to the solution of this problem lies in the coating of this hypothenuse with a protecting film of high refractive index. This suggestion is based on the observation that, when the hypothenuse is coated with a film of stibnite ($n=4.0$) the line of demarcation remains sharp under all conditions. Stibnite is too soft, however. Finally a film of silicon ($n=4.2$) was "sputtered"[3] and baked at 125° for 12 hours. This film was fairly hard. It is here suggested that instrument-makers either develop a new, high-refractive-index glass which resists the formation of surface films, or coat the present vulnerable prism with a thick, hard film of TiO_2 ($n=2.4$).

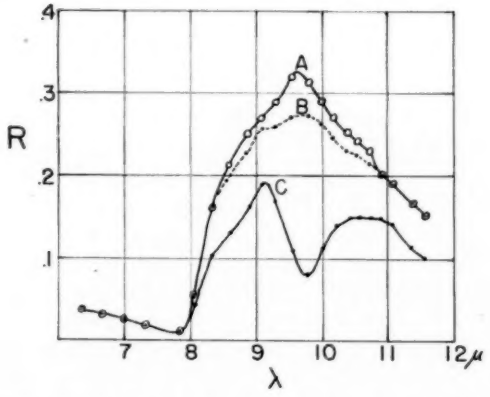

FIG. 6. A is the reflection curve for bare glass ($n=1.52$), B is the reflection curve for a Nicoll skeleton film ($nt=1300$A), C is the reflection curve for a Nicoll skeleton film ($nt=6500$A).

[3] E. O. Hulburt, Astrophys. J. **42**, 205 (1915).

IV. OPTICAL PROPERTIES OF SILICA SKELETON FILMS ON GLASS

Closely related to the previous work is that of Nicoll.[4] He showed that certain constituents of a glass surface are more readily attacked by vapors of hydrofluoric acid than others. If, after exposure to HF vapors, the plate is washed, there remains a "skeleton-film"[5] of amorphous silica which, at proper thickness, renders the surface "non-reflecting." His work establishes the two points: (1) the film is composed of amorphous silica;[6] (2) the refractive index of the film "is slightly less than 1.3." In view of the definiteness of these results, it seemed of interest to determine whether or not the procedures used by the writer would lead to results in agreement with those of Nicoll.

Dr. Nicoll kindly prepared for the writer two glass plates ($n=1.52$) coated with skeleton films of optical thickness, $nt=1300$ and 6500A, respectively. As before, infra-red reflection characteristics of these films were studied. The results are presented in Fig. 6. It is evident that the reflection curve for the heavier film (C) differs markedly from that of the original glass surface (A). The principal reflection maximum for curve (C) at 9.1μ again lies close to that of fused silica at 9.25μ. That the reflection maximum is shifted slightly toward shorter wave-lengths is attributed to the fact that the film is skeletonized instead of being homogeneous. The conclusion drawn from these observations is that film in question is, indeed, amorphous silica.

As for the refractive index of the film, the problem was attacked according to the procedure developed by the writer in his study of "The Colors of Mother of Pearl."[7] At the time when this work was carried out, a formula for the determination of the refractive index of the minute laminae in mother of pearl was presented. This formula has the great advantage of not requiring a knowledge of the thickness of layer. Because of its usefulness the derivation of the formula is here (belatedly) presented:

If white light is reflected from a surface-film of considerable thickness and if this light be

[4] F. H. Nicoll, R.C.A. Rev. **6**, 287 (1942).
[5] K. B. Blodgett, Phys. Rev. **55**, 391 (1939).
[6] F. H. Nicoll and F. E. Williams, J. Opt. Soc. Am. **33**, 434 (1943).
[7] A. H. Pfund, J. Frank. Inst. p. 462 (April 1917).

analyzed by means of a spectroscope, alternate inteference maxima and minima (the so-called "channeled spectrum") will be observed (Fig. 7). If the angle of incidence be increased, the fringes move toward shorter wave-lengths. Recalling that the retardation Δ between two interfering beams is:[8]

$$\Delta = 2nt \cos r = m\lambda.$$

Let

λ_1 = wave-length of maximum for incidence angle i_1,

λ_2 = wave-length of maximum for incidence angle i_2.

Then

$$2nt \cos r_1 = m\lambda_1,$$

$$2nt \cos r_2 = m\lambda_2.$$

Dividing:

$$\cos r_1 / \cos r_2 = \lambda_1/\lambda_2. \quad (1)$$

But

$$\cos r = (1 - \sin^2 i/n^2)^{\frac{1}{2}}.$$

Substituting in (1), we find

$$n = \left(\frac{\lambda_2^2 \sin^2 i_1 - \lambda_1^2 \sin^2 i_2}{\lambda_2^2 - \lambda_1^2} \right)^{\frac{1}{2}}. \quad (2)$$

[8] Jenkins and White, *Fundamentals of Physical Optics* (McGraw-Hill Book Company, Inc., New York, 1937), first edition, p. 84.

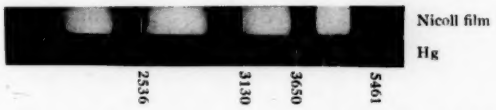

FIG. 7. The continuous spectrum of a H_2 vacuum tube is reflected from a Nicoll film of optical thickness 6500A. Note alternate interference maxima and minima.

To utilize this formula, the light of a H_2 vacuum tube was reflected from a Nicoll film ($nt = 6500$A) into a small quartz spectrograph at the angles of incidence 11° and 45°, respectively. Since interference minima instead of maxima may be used (so long as the order of interference m remains the same), use was made of the former. By making the times of exposure suitably long, the widths of the minima are much reduced. By means of a comparator the wave-lengths of the minima were determined. Pertinent data are:

$$\lambda_2 = 0.53\mu, \quad i_2 = 11°,$$

$$\lambda_1 = 0.45\mu, \quad i_1 = 45°.$$

According to calculations based on formula (2), it is found that $n = 1.29$. This again corroborates the statement of Nicoll that the refractive index of his film "is slightly less than 1.30."

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A Combination Arc-Spark Source for Magnesium Analysis*

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THIS paper describes a combined arc and spark source with circuit components chosen for magnesium analysis. The basic circuit is one of great versatility, so two distinct sources can be built with many components in common. This makes it advantageous to incorporate both sources in one unit. Significant advantages over the usual high voltage spark and continuous arc are obtained.

The arc source differs from the conventional types in that it is interrupted periodically. It was developed for use in analyzing magnesium alloys. When the usual continuous arc[1] is used, the arc streamer seldom wanders over the surfaces of a pair of magnesium electrodes in such a way that it samples the whole surface. Instead, it settles upon a spot of glowing oxide fairly early in the exposure and causes a large amount of continuous radiation. The amount of this continuous radiation varies widely from sample to sample.

Movement of the arc streamer to a new location can be secured by extinguishing the arc, allowing the oxide to cool, and restarting it. It is

* Paper presented at the 30th Annual Meeting of the Optical Society of America, New York, New York, October 18–20, 1945.

[1] O. S. Duffendack and K. B. Thomson, Proc. A. S. T. M. **36**, Part II, 301–309 (1936).

not difficult to restart the arc a large number of times per exposure by means of a high frequency spark.[2,3] The remaining question, then, is how often to stop and start the arc and what percentage of the time it is to be arcing.

The rate of starting and stopping the arc must be kept fairly low to allow time for sufficient cooling. Interruption rates of one to ten per second have been tried, and the arc current has been allowed to flow from one-half to one-hundreth of a period. When the arc current was allowed to flow only a short time in each period, the instantaneous current used was made higher so that the source would have a reasonable intensity. The pulse of current was obtained either directly from the alternating current line or from a large condenser (320 μf) charged to 250 volts.[4]

Figure 1 shows the basic circuit. By the use of different circuit constants a wide variety of excitation conditions can be obtained. These conditions may be subdivided into two groups: first, the excitation is caused primarily by current drawn through r during the discharge; and second, the excitation is caused primarily by the discharge of the capacitor, C.

In the first case C is made small (0.01μf) and serves as a by-pass for the high frequency spark. The current drawn through r may be either alternating or direct. If it is alternating and rather small, the arc will be self-extinguishing when the voltage passes through zero. If it is alternating and large, or direct current, some way must be provided to stop the current when the discharge is to be extinguished. This source is arc-like.

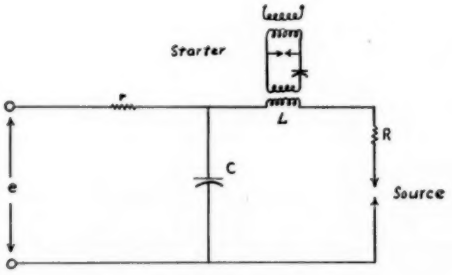

FIG. 1. Basic circuit.

In the second case C is made large ($\frac{1}{2}$ μf or more). The supply voltage may be alternating or direct. The charging resistor r is chosen so that C can be charged up nearly to the supply voltage and yet allow the source to draw a minimum of current through r during the discharge. The discharge is self-extinguishing. The character of the source depends on whether the values of R, C, and L give an oscillatory, critically damped, or over-damped discharge, but in general it is spark-like.[5]

The experimental units showed that sensitivity for trace elements, particularly Fe, was less for a source which made use of short but rather high current pulses than for the continuous arc at lower currents. The conditions chosen were therefore such as to give a source as much like a continuous arc as possible and still make use of the interrupted feature and its advantage of more arc streamer movement. The arc is maintained for $\frac{1}{2}$ second out of each second at 4 amperes. This allows sufficient time for the oxide to cool on magnesium pins of 3-mm diameter or larger so that the arc strikes to a new place on the electrode tips each time. The current value is made $1\frac{1}{2}$ times that used in the continuous arc to compensate for the loss in exposure which results from having the source on only 50 percent of the time. This small increase does not noticeably reduce the sensitivity.

The spark source is an outgrowth of the experimental work mentioned above. When most of the excitation was secured from the charge in the condenser, the source was definitely spark-like even though the condenser was charged to a rather low voltage. It showed excellent reproducibility and gave promise of eliminating two troublesome features of the synchronous high voltage spark. First, since most of the power is obtained at moderate voltage, leakage becomes unimportant. Changes in absolute humidity had caused shifts in the analytical calibration for the high voltage spark previously used. Second, the low voltage spark causes less interference with other sensitive instruments in the building since it generates and radiates less radiofrequency power.

Figure 2 gives the wiring diagram of the com-

[2] Henry Hemmendinger, J. Opt. Soc. Am. **32**, 149 (1942).
[3] J. L. Saunderson and V. J. Caldecourt, J. Opt. Soc. Am. **34**, 116 (1944).
[4] K. Pfeilsticker, Zeits. f. Metallkunde **30**, 211 (1938).
[5] M. F. Hasler and H. W. Dietert, J. Opt. Soc. Am. **33**, 218 (1943).

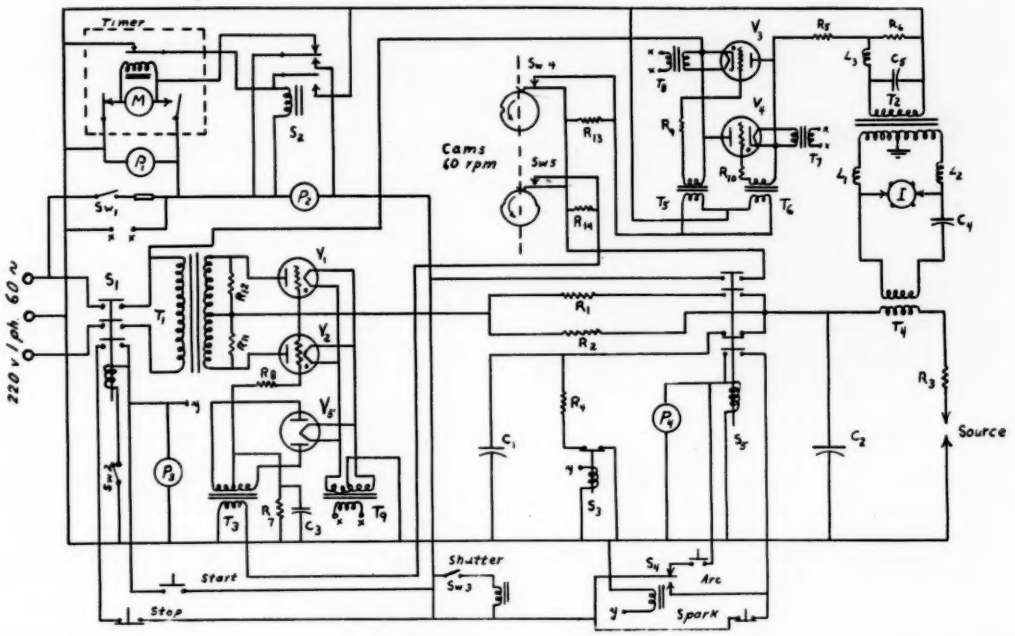

FIG. 2. Combined source unit, parts list.

C_1	—0.25 μf; 4000 v
C_2	—0.01 μf; 4000 v
C_3	—0.25 μf; 600 v
C_4	—0.002 μf; 20,000 v
C_5	—0.1 μf; 600 v
I	—Interrupter 1800 r.p.m. synchronous
L_{1-3}	—R.F. Chokes
P_{1-4}	—Indicator lamps
R_1	—550 ohms; 10,000 watts
R_2	—5000 ohms; 400 watts
R_3	—Damping resistor
R_4	—500 ohms; 20 watts
R_5	—13 ohms; 1500 watts
R_6	—80 ohms; 600 watts
R_{7-10}	—50,000 ohms; 1 watt
$R_{11,12}$	—100,000 ohms; 200 watts
$R_{13,14}$	—10,000 ohms; 1 watt
S_1	—Contactor, 100 amp., 220-v contacts
S_2	—Relay, 10 amp., 110-v contacts
S_3	—Contactor, Allen and Bradley, 10 amp., 600-v a.c. contacts
S_4	—Relay, 10 amp., 110-v contacts
S_5	—Contactor, Allen and Bradley, 10 amp., 600-v a.c. contacts
S_{w1}	—Main switch
S_{w2}	—Interlock for source (cabinet door)
S_{w3}	—Shutter switch
S_{w4}	—Starter control
S_{w5}	—Arc control
T_1	—220/2200-v, 10 kva line transformer
T_2	—110/15,000-v, 450 volt-amperes, luminous sign transformer
T_3	—110/220-v isolating transformer
T_4	—r-f transformer 11T 3½" diam., 4" long 44T 2½" diam., 4" long
$T_{5,6}$	—110/6-v, 1.5 amp. filament transformer
T_{7-9}	—110/5-v filament transformer
Timer	—G.E. TSA-10
$V_{1,2}$	—Thyratron KU677
V_5	—Type 80
$V_{3,4}$	—Thyratron FG57

plete source unit. Power for both sources is obtained from the main transformer T_1 through the full wave grid controlled rectifiers, $V_{1,2}$. With S_5 de-energized as shown, the unit is in the spark position. Then condenser C_1 of 0.25 μf capacity is charged to nearly 3000 volts through R_2. To release this charge a synchronous arc starter is used. The arc starter utilizes a synchronous interrupter adjusted so that a discharge occurs at the interrupter for each peak of the 60-cycle alternating current supply. T_4 thus introduces a pulse of high frequency high voltage into the source circuit 120 times per second. Each time this pulse causes a spark at the sample electrodes, C_1 discharges through the gap. This discharge occurs when the voltage across the starter transformer is at a maximum and likewise the capacitor C_1 has reached its maximum voltage. The discharge may be oscillatory, critically damped, or over-damped, depending on the size of R_3. We have two spark units and are using 1-ohm added resistance in one unit and no added resistance in the other. Sufficient resistance for critical damping and slight over-damping has been used also. There appears little choice as to which is the best. Even with no added resistance the frequency of the oscillatory discharge is low, 15–20 kc, which accounts for the small amount of troublesome r-f radiation. Power consumption for T_1 when supplying the spark is slightly over 500 watts. Since this load is full wave and fairly small, it is feasible to provide constant voltage for the spark with a constant voltage transformer.

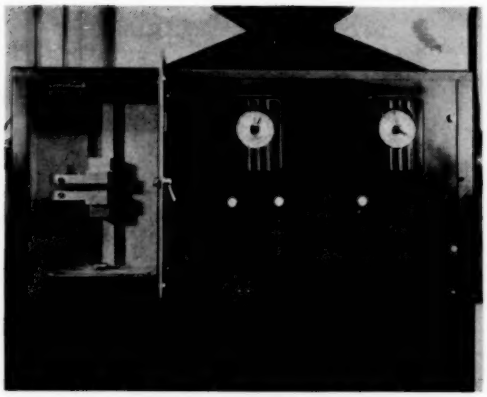

FIG. 3. Source unit in use with the direct-reading spectrometer.

By energizing S_5, the source unit is ready for arcing. The controlled rectifiers will provide up to 8 amperes through the resistance bank R_1. The arc starter is controlled by the cam-operated switches S_{w4} and the thyratrons $V_{3,4}$. The starter provides a series of high voltage pulses for about 0.1 second which ignites the arc. At this time $V_{1,2}$ are conducting. After the arc has burned 0.5 second, $V_{1,2}$ are made non-conducting by S_{w5} which is cam-operated also. The source is off 0.5 second and then the cycle repeats. C_2 serves as an r-f by-pass and does not appreciably affect the spectra. A current of 4 amperes is drawn while the arc is running.

The timer and relay S_2 are used to guarantee correct filament warm-up for $V_{1,2}$. When the unit is first turned on, or after a power failure, the source cannot be started until timer M completes its cycle of 5 minutes.

Both sources have been in routine use for the analysis of magnesium alloys for nearly two years in the photographic spectrographic laboratory and the spark alone for nearly one year with the direct-reading spectrometer.[6] They have proved successful in minimizing the faults found in our previous sources. In the case of the spark, no calibration shifts have occurred in the two years of operation. The r-f interference has been considerably less than with the synchronous high voltage spark, and the reproducibility appears to be the same. The interrupted arc has shown a much more reproducible background from spectrum to spectrum and somewhat less effect of high Zn and Al upon the intensity of the background than the conventional continuous arc. The pins run cooler which permits smaller pins or thinner sheet to be arced without serious burning.

Since the circuit allows one to select an arc or spark and to start or stop each by push-button control, the source unit is well suited for rapid operation by non-technical personnel. It is also simple to add electric timers for automatic control. Figure 3[7] shows the arc-spark unit used with our direct-reading spectrometer. The controls and electric timers which determine the pre-exposure and exposure periods are on the panel on the right. The electrode clamps and shutter are incorporated in the source cabinet also. The stack topped by an air filter which is visible at the upper edge of the picture is the exhaust for the forced air used to cool the resistance bank and ventilate the source compartment.

[6] J. L. Saunderson, V. J. Caldecourt, and E. W. Peterson, J. Opt. Soc. Am. (to be published).

[7] This source unit was built for us by Baird Associates.

On the Primary Chromatic Coefficients of a Lens System

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Expressions are derived for the primary longitudinal and transverse chromatic aberrations of a lens system as summations of terms over all the singlet components of the system. Each term in these summations is the product of the partial dispersion of the glass of the lens with a certain coefficient which is easily calculated from the data of a paraxial trace, and specifies the contribution made by the lens to the corresponding primary chromatic aberration of the final image. The coefficients are called the primary chromatic coefficients of the system. In addition, the coefficients provide a rapid means of analyzing the complete secondary spectrum of the system, and afford a clear guide to the adjustment of the primary chromatic aberrations insofar as it is desired to do this by the selection of different glass types.

IN a discussion of the theory of the primary chromatic aberration of a lens system Conrady[1] arrives at expressions for the contributions made by a surface of the system to the longitudinal and transverse chromatic aberrations of the final image having the forms

$$LchC' = \frac{N'u'l'i'}{N_k'u_k'^2}\left(\frac{\delta N'}{N'} - \frac{\delta N}{N}\right), \tag{1}$$

$$TchC' = \frac{N'u'l'i_{pr}'}{N_k'u_k'}\left(\frac{\delta N'}{N'} - \frac{\delta N}{N}\right) \tag{2}$$

$$= LchC'u_k'\left(\frac{i_{pr}'}{i'}\right).$$

The total aberration of each kind is then given by the sum of the contributions from all the surfaces of the system. To compute these quantities the traces of an axial paraxial ray and a principal paraxial ray are required. It is proposed to show now that there are advantages to be gained by considering the contributions made by the separate *components* of the system rather than the contributions of the separate surfaces, a component being any singlet lens in the system. The desired expressions may be obtained by algebraic manipulation of Eqs. (1) and (2), but another development is given on account of the insight which it affords into the significance of the final expressions and as illustrating a method which has fruitful applications in optical theory.

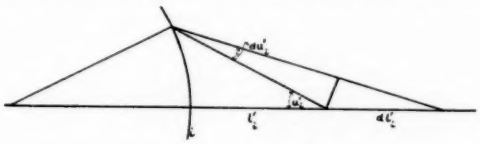

FIG. 1.

[1] A. E. Conrady, *Applied Optics and Optical Design* (Oxford University Press, London, 1929), pp. 312–313.

1. THE EFFECT OF SMALL CHANGES IN REFRACTIVE INDEX ON THE FINAL INTERSECTION LENGTH OF A PARAXIAL RAY

Consider a ray from an axial object point the path of which is traced paraxially through the lens system. The ray emerges from the last surface of the system with an intersection length l_k' and an inclination angle u_k'. At the ith surface of the system the ray is incident with an intersection length l_i and an inclination angle u_i, and its path after refraction at the surface is specified by the corresponding quantities l_i' and u_i'. Suppose now that a small change of refractive index of amount dn_i is made at this surface, where $n_i = N_i/N_i'$. As a result of this change the ray after refraction will follow a new path which may be specified by the quantities $l_i'+dl_i'$ and $u_i'+du_i'$. From Fig. 1 it is easily seen that

$$dl_i' = -l_i'du_i'/u_i'. \tag{3}$$

Again, since

$$u' = u+i-i' = u+i(1-n),$$

we have

$$\partial u'/\partial n = -i, \tag{4}$$

and Eq. (3) becomes

$$dl_i' = -l_i'\frac{\partial u_i'}{\partial n_i}dn_i \Big/ u_i' = l_i'i_idn_i/u_i'. \tag{5}$$

As a result of the change made at surface i, the ray will emerge from the last surface of the system along a new path specified by $l_k'+dl_k'$ and $u_k'+du_k'$. The change, dl_k', in the intersection length may be expressed in terms of dl_i' by means of the longitudinal magnification law which gives

$$N_k'dl_k'u_k'^2=N_i'dl_i'u_i'^2$$

and combining this with Eq. (5) we obtain

$$dl_k'=\frac{N_i'u_i'l_i'i_i}{N_k'u_k'^2}dn_i,$$

whence

$$\begin{aligned}\partial l_k'/\partial n_i&=N_i'u_i'l_i'i_i/N_k'u_k'^2\\&=N_i'y_ii_i/N_k'u_k'^2.\end{aligned}\tag{6}$$

Now consider the general component, h, of the system which is any singlet lens. We shall use the subscripts 1 and 2 to denote the first and second surfaces of this component. The refractive index of the component is N_h, while the media on either side of it have refractive indices N_{h-1} and N_{h+1}, respectively. Suppose that the refractive index of the component is changed to the value N_h+dN_h. This change involves n-changes of different amounts at the two surfaces of the component, and their effect on the path of the ray entering the final image space can be determined with the aid of Eq. (6). Thus

$$\begin{aligned}\frac{\partial l_k'}{\partial N_h}&=\frac{\partial l_k'}{\partial n_1}\frac{\partial n_1}{\partial N_h}+\frac{\partial l_k'}{\partial n_2}\frac{\partial n_2}{\partial N_h},\\&=\frac{\partial l_k'}{\partial n_1}\frac{\partial}{\partial N_h}\left(\frac{N_{h-1}}{N_h}\right)+\frac{\partial l_k'}{\partial n_2}\frac{\partial}{\partial N_h}\left(\frac{N_h}{N_{h+1}}\right),\\&=-\frac{N_1'y_1i_1}{N_k'u_k'^2}\cdot\frac{N_{h-1}}{N_h^2}+\frac{N_2'y_2i_2}{N_k'u_k'^2}\cdot\frac{1}{N_{h+1}},\\&=\frac{1}{N_k'u_k'^2}(-n_1y_1i_1+y_2i_2),\\&=\frac{1}{N_k'u_k'^2}(-y_1i_1'+y_2i_2).\end{aligned}\tag{7}$$

In the next section we shall use Eq. (7) in the derivation of an expression for the primary longitudinal chromatic aberration, but it is of considerable interest in itself as affording a simple expression for the rate of change of the back focal length of the system with the refractive index of the glass of any singlet lens of the system. Its form is easily remembered as the incidence angles are those within the glass of the lens concerned, while the negative sign goes with the dashed angle.

2. THE PRIMARY LONGITUDINAL CHROMATIC ABERRATION

Equation (7) may now be used to investigate the paraxial chromatic properties of the lens system if, instead of thinking of glass changes at the various components, we consider the passage through the system of rays of different colors. The traced paraxial ray gives the path through the system for a ray of some intermediate color d, which very commonly may be sodium light. To change from this ray to a ray of red light, r, incident on the system along the same path in the initial medium, only involves a refractive index change from N_d to N_r at each component. Hence we may write

$$\begin{aligned}l_{rk}'&=l_{dk}'+\frac{\partial l_k'}{\partial N_a}(N_r-N_d)_a+\frac{\partial l_k'}{\partial N_b}(N_r-N_d)_b+\cdots\\&=l_{dk}'+\sum\frac{\partial l_k'}{\partial N_h}(N_r-N_d)_h.\end{aligned}\tag{8}$$

Similarly, for a ray of color v at the other end of the spectrum

$$l_{vk}'=l_{dk}'+\sum\frac{\partial l_k'}{\partial N_h}(N_v-N_d)_h.\tag{9}$$

If the colors r and v are those for which the system is to be achromatized, then the longitudinal chromatic aberration is given by

$$lch'=l_{rk}'-l_{vk}'=\sum\frac{\partial l_k'}{\partial N_h}(N_r-N_v)_h.\tag{10}$$

This equation is of considerable value as giving

TABLE I.

Component	$\partial l_k'/\partial N$	N_D	V
a	−587.858	1.6151	55.3
b	625.028	1.5491	46.7
c	−155.558	1.5185	60.4
d	−114.533	1.5185	60.4
e	459.352	1.5491	46.7
f	−403.975	1.6151	55.3

TABLE II. Detailed computation of the contributions to the primary longitudinal chromatic aberration made by the three pairs of components of a wide angle Ross Xpres type photographic objective for various spectral ranges.

Comp.	chr. coeff.	N_b-N_D	Contrbn.	N_C-N_D	Contrbn.	N_e-N_D	Contrbn.	N_F-N_D	Contrbn.	$N_{G'}-N_D$	Contrbn
a, f	−991.833	−0.00517	5.1278	−0.00328	3.2532	0.00274	−2.7176	0.00785	−7.7859	0.01422	−14.1039
b, e	1087.38	−0.00539	−5.8610	−0.00342	−3.7188	0.00291	3.1643	0.00835	9.0796	0.01529	16.6260
c, d	−270.091	−0.00406	1.0966	−0.00256	0.6914	0.00211	−0.5699	0.00603	−1.6286	0.01089	− 2.9413
		$lch_{bD'}=0.3634$		$lch_{CD'}=0.2258$		$lch_{eD'}=-0.1232$		$lch_{FD'}=-0.3349$		$lch_{G'D'}=-0.4192$	

the chromatic aberration of a lens system as a sum of terms taken over all the components of the system, each term being the product of a certain differential coefficient with the dispersion of the glass of that component. We shall call these differential coefficients, $\partial l_k'/\partial N_h$, the primary longitudinal *chromatic coefficients* of the system. It is obvious that each term in the summation in Eq. (10) represents the contribution of one component to the final chromatic aberration. Hence, if the dispersion of the glass of one component is altered, the resultant effect on the chromatic aberration is easily determined. Writing $(N_v-N_r)_h=P_h$ we have

$$lch=-\sum\frac{\partial l_k'}{\partial N_h}P_h,$$

and hence

$$\partial lch/\partial P_h=-\partial l_k'/\partial N_h. \qquad (11)$$

Thus the chromatic coefficients specify the rate of change of the back focal length of the system with the refractive index of each component, and also give the negative rate of change of the longitudinal chromatic aberration with the dispersion of each component. It should be noticed further that once the chromatic coefficients of the system are calculated for a paraxial ray of some intermediate color it is possible to derive very quickly the final intersection length of a paraxial ray of any color from the same object point. Hence we can obtain complete and accurate information of the variation of the chromatic correction throughout the whole spectrum, or, in other words, an accurate analysis of the secondary spectrum of the system.

3. A NUMERICAL EXAMPLE

We will consider an example now in order to give an idea of the order of accuracy of the chromatic coefficients and also to show the type of question to which they provide a rapid answer. A paraxial trace was made through a rough design of a photographic objective of the wide angle Ross Xpres type, and the chromatic coefficients of the six components were calculated. In Table I the calculated values of the coefficients and the constants of the glasses are listed. Since the same glass is used in components a and f, we may combine the chromatic coefficients of these two components, for in the summation of Eq. (10) each coefficient will be multiplied by the same dispersion. The same holds for components b and e, and for components c and d. Hence we have

$$\partial l_k'/\partial N_{a,f}=-991.833,$$
$$\partial l_k'/\partial N_{b,e}=\ 1087.38,$$
$$\partial l_k'/\partial N_{c,d}=-270.091.$$

In Table II the details of the computation of the contributions of the components to the chromatic aberration, and the total chromatic aberration, of the system for various spectral ranges are set out. In the first place consider the general accuracy of the results obtained by the use of the coefficients. From the trace we have $l_k'=106.525$, and considering the spectral range $C-F$, for example, Eqs. (8) and (9) together with some results from Table II give at once

$$l_{Ck}'=106.525+0.226=106.751,$$
$$l_{Fk}'=106.525-0.335=106.190,$$
$$lch_{CF}'=0.561.$$

A full trace of the system for paraxial rays of colors C and F gives

$$l_{Ck}'=106.748,$$
$$l_{Fk}'=106.181,$$
$$lch_{CF}'=\ \ 0.567,$$

from which it will be seen that the chromatic coefficients give predictions of the final intersection lengths of rays of other colors, and hence

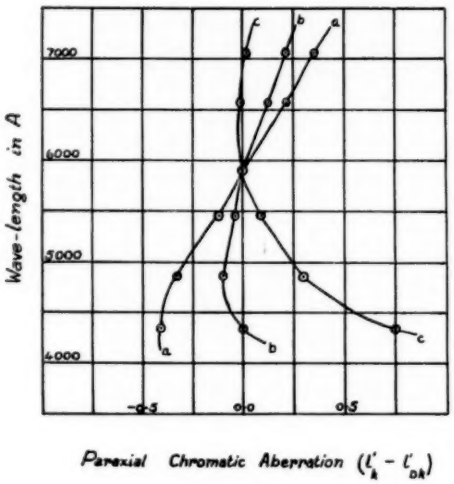

FIG. 2.

of the chromatic aberration, to a very valuable order of accuracy.

In the results of Table II we have essentially an analysis of the secondary spectrum of the system. By plotting the values of $l_k' - l_{Dk}'$ against the wave-length of the spectral line concerned we summarize the chromatic correction state of the system by the line *aa*, which describes a very undesirable state of affairs. The chromatic coefficients may be used next to indicate the way in which improvement may be obtained. A study of Table II shows that the contributions from components *a* and *f* are too great, or alternatively, the contributions from components *b* and *e* are too small. Improvement may be obtained by replacing the glass of *a* and *f* by another DBC of higher *V*-number, or alternatively, by replacing the flint glass of components *b* and *e* by one of lower *V*-number. Calculating the effect of the first possibility by changing to a DBC of the type $N_D = 1.6132$, $V = 56.8$, we show in Table III the computation of the new secondary spectrum values. The resulting chromatic correction state is exhibited in the curve *bb* in Fig. 2, which shows that the foci for *D* and *G'* have been brought together giving achromatism of the usual photovisual type. A change to a DBC glass with a still higher *V*-number will produce another type of achromatism. Thus if we use the glass $N_D = 1.6134$, $V = 59.3$, we get a correction state summarized by the curve *cc* in Fig. 2, a type of achromatism which would be suitable over the spectral range from 7000A to 5100A, corresponding to the use of a panchromatic emulsion with a K 2 or blue minus filter. Smaller changes could be made by suitable choice of glass for components *c* and *d*, for which the chromatic coefficients are smaller, but the available range of glasses is somewhat more restricted. The example provides, however, sufficient evidence of the power of the chromatic coefficients in analyzing the chromatic correction and directing the choice of glasses for achromatism of a desired type.

4. THE PRIMARY TRANSVERSE CHROMATIC ABERRATION

Suppose that a principal paraxial ray for some given obliquity has been traced through the lens system. After refraction at surface *i*, this ray intersects the paraxial image plane behind this surface (i.e., a plane at right angles to the principal axis distant l_i' from the pole of the surface) at a point of which the height above the axis is h_i'. If a change of refractive index of amount dn_i were made at this surface, the principal paraxial ray would undergo a change of direction du_{ipr}' after refraction, and would intersect the paraxial image plane at a height $h_i' + dh_i'$ such that

$$\begin{aligned} dh_i' &= -l_i' du_{ipr}', \\ &= -l_i' \frac{\partial u_{ipr}'}{\partial n_i} dn_i, \\ &= l_i' i_{ipr} dn_i. \end{aligned} \tag{12}$$

If the traced ray intersects the final paraxial image plane at a height h_k' above the principal

TABLE III. Computation of the change of the secondary spectrum, $l_k' - l_{Dk}'$, due to change of glass in two components of the system.

Partial dispersion, $P = N - N_D$	dP	$d(l_k' - l_{Dk}')$	New $(l_k' - l_{Dk}')$
−0.00502	0.00015	−0.1488	0.2146
−0.00318	0.00010	−0.0992	0.1266
0.00266	−0.00008	0.0793	−0.0439
0.00762	−0.00023	0.2281	−0.1068
0.01380	−0.00042	0.4166	−0.0025

axis of the system, the change dn_i will produce a shift of this intersection point through a distance dh_k' which is given by the Lagrange magnification law as

$$dh_k' = N_i' u_i' dh_i' / N_k' u_k',$$

which on combination with Eq. (12) leads to the relation

$$\partial h_k' / \partial n_i = N_i' y_i i_{ipr} / N_k' u_k'. \qquad (13)$$

If we wish to relate this derivative to the corresponding longitudinal one, we have

$$\partial h_k' / \partial n_i = u_k' (i_{ipr} / i_i)(\partial l_k' / \partial n_i). \qquad (14)$$

If the refractive index of the general component, h, is changed by an amount dN_h, then by analogy with the derivation of Eq. (7) we have

$$\frac{\partial h_k'}{\partial N_h} = \frac{1}{N_k' u_k'}(-y_1 i_{pr1}' + y_2 i_{pr2})_h, \qquad (15)$$

the subscripts 1 and 2 referring to the first and second surfaces of component h. Equation (15) provides the most convenient form for the computation, requiring less operations than the alternative use of Eq. (14). In precise analogy to Eq. (10) we may write down an expression for the transverse chromatic aberration as

$$tch' = h_{rk}' - h_{vk}' = \sum \frac{\partial h_k'}{\partial N_k}(N_r - N_v)_h. \qquad (16)$$

It should be noticed that the intersection height, h_{rk}', for example, is not the intersection height of the principal paraxial ray for red light in the paraxial image plane for red light, but it is the height at which a red ray entering the system along the traced path of the principal ray for the intermediate color d intersects the plane at right angles to the principal axis through the paraxial focus for the color d. This does not affect the validity of our expression for tch', for Eq. (17) still measures the separation, in a fixed plane through the mean paraxial focus, of the intersection points of two principal paraxial rays of the colors r and v. Each term in the summation of Eq. (16) gives the contribution to the transverse chromatic aberration of the final image made by one component of the system. The quantities $\partial h_k' / \partial N_h$ are called the primary transverse chromatic coefficients of the system. Their computation requires very little work, only four operations for each component, which is considerably less than the surface contribution analysis given by Conrady. Once these coefficients have been computed, the variation of the correction state of the system in respect of primary transverse chromatic aberration may be examined quickly throughout the spectrum. A study of the values of the coefficients allows the designer to vary his selection of glasses to give the best state of correction for this aberration in exactly the same way as was exemplified in the case of the longitudinal aberration. If the surface contributions are required in any particular case, they are easily obtained as in both Eqs. (7) and (15); the two terms entering into the expressions each have reference to a single surface. The extraction from the computation of the terms referring to each surface, one in number for a glass-air interface and two for a contact surface, permits the calculation of the surface contributions. The transverse chromatic coefficients are calculated for the particular obliquity at which the principal paraxial ray has been traced, while those for other obliquities vary in direct proportion to the obliquity.

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Measurement of the Radius of Curvature of Concave Spheres

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THE method of measuring the radius of curvature of a sphere with an autocollimating microscope using a Gauss eyepiece is well known. The use of the Gauss eyepiece suffers from several practical disadvantages. The condition that one has to hunt for one bright field on another bright field in the preliminary adjustment at the center of curvature is both difficult and vexatious. Location of the surface of the sphere is accomplished by distributing some very fine foreign material over the surface of the sphere and locating the surface by bringing these fine particles into sharp focus in the microscope. An alternative method is to watch for the point at which the cross hairs become sharpest as the microscope is moved toward the surface causing the real cross hairs and their reflected image to coalesce. This latter operation is very hard to achieve with high precision due to the fact that the cross lines are very narrow and located on a bright field.

Substitution of an Abbe eyepiece for a Gauss eyepiece on the microscope leads to a considerable improvement in ease of manipulation in making precise radius of curvature measurements. At the center of curvature one hunts for a brilliant spot in a dark field in making the preliminary adjustment. The microscope can be very considerably out of focus, and it is still possible to find the bright patch even though the adjustment is so poor that the out-of-focus reflected image of the prism face nearly covers the entire field of the eyepiece. Upon finding the reflected image it is of course very simple to complete the final focusing by means of suitable translatory motions.

Abbe eyepieces of the type manufactured by the Gaertner Scientific Company have the following pertinent characteristics. The illuminating prism has a rectangular face toward the objective of the microscope. The cross hairs which are illuminated by the light emerging from the prism face are located about 1.7 mm nearer to the objective than said face. Face of the illuminating prism is ground and polished to sharp edges. The edges of the prism "frame" the illuminated face making it act as though it was the source of light.

1. MEASUREMENT OF REFLECTED IMAGE AT THE CENTER OF CURVATURE

For determination of radius of curvature it is required to know the position of the image of the illuminating prism face when said image produced by the microscope objective is exactly at the center of curvature of the sphere. Since this image will be made use of twice, once at the center of curvature, and then on the surface of the sphere, the distance desired is the difference between the settings of the microscope, for the two above-mentioned critical positions. (Absolute relationship of the image of the prism face to the front surface of the microscope objective is of no interest for concave spheres.) If the cross lines were etched on the face of the illuminating prism the first desired critical position at the center of curvature would be the position of best focus of these cross lines. For practical reasons it is better to focus on the dark cross lines in the uniformly illuminated bright field of the prism than on the less well-defined edges of the prism. Since the cross lines are located somewhat towards the objective compared to the prism face the image of these cross lines will be nearer to the surface of the sphere than the image of the illuminating prism face.

Let us call the distances between cross hairs and prism face dp and dq, respectively, in object and image space of the microscope objective. M is the magnification of the microscope objective. It is easily shown that

$$dq = dp/M^2, \tag{1}$$

with a $dp = 1.7$ mm for common objectives dq, has the following values:

$3X = .189$ mm $\quad 10X = .017$ mm $\quad 21X = .0038$ mm.

Focusing the microscope on the reflected image

of the dark cross hairs in the illuminated slightly out-of-focus image of the prism face serves a second useful purpose. If the numerical aperture of the microscope is large enough so that the whole sphere is illuminated the sharpness and blackness of the cross hair image gives a good indication of the sphericity of the surface under test.

2. MEASUREMENT OF SURFACE POSITION

When the microscope is moved into position so that the image of the illuminating prism face produced by the objective falls on the surface of the sphere nearly in focus it will be noticed that a halo appears around the edges of the illuminating prism when viewed from the back by means of the eyepiece. As the image comes more nearly in focus on the surface, the halo decreases in size and finally disappears when true focus on the surface is reached. On passing through the focus the halo reappears. The point at which this halo disappears is extremely sharp if the prism is ground to sharp edges. This halo is obviously caused by an out-of-focus image of the illuminated face. This image will exactly coincide with the real face by virtue of reflection at the surface of the sphere when true focus on the surface is reached. The light returning from the surface of the mirror does not include all the rays reflected from the mirror since the rays from the extended image by virtue of the law of reflection return through the objective lens through different parts of the lens than on their first transmittal. Some reflected rays miss the objective completely and are lost. A test was made as to the exactness of the disappearance of the halo with coincidence of the focus on the surface of a sphere in the following fashion. The microscope was equipped with a $43X$, 4-mm objective. The power was sufficient that with proper illumination microscopic polishing scratches on the surface could be seen. It was found that coincidence of "blackout" (disappearance of halo) and best focus on polishing scratches was simultaneously achieved. Since the sensitivity of the eyepiece end of the microscope is the same for all objectives (N.A. eyepiece end constant) regardless of N.A. of objective at constant tube length the limitation on sensitivity of an objective on "blackout" end point is something approaching the focal range of the objective if well-corrected.

The true radius of curvature is obtained as follows. Position of the microscope when in focus on the cross hairs at the center of curvature position minus the "blackout" position minus dq.

Sensitivity of Measurement

If two concave spheres of slightly different radius of curvature are placed in contact at their edges, the difference in sagittal depth necessary to cause a detectable change in the character of the image produced at the center of curvature will be $\lambda/8$ applying the Rayleigh criterion. To a first-order approximation

$$\Delta S = r^2 \Delta R / 2R^2, \tag{2}$$

where r is half the diameter of the sphere, ΔS the difference in sagittal depth of the two curves, and R the radius of curvature. Substituting the Rayleigh limit in the above equation

$$R = \pm \frac{\lambda}{4} \csc^2 \alpha, \tag{3}$$

where α is the semi-angular aperture of the sphere. If the same reasoning is applied to the spherical wave front emanating from a lens, the focal range F[1] becomes

$$F = \pm \frac{\lambda}{2} \csc^2 \alpha, \tag{4}$$

where α is the analogous semi-angular aperture of the lens in question.

The error in making a radius determination will be of the order of magnitude of $\Delta R + \Delta F = \Delta R_e$

$$\Delta R_e \sim \frac{3\lambda}{4} \csc^2 \alpha. \tag{5}$$

Equation (5) is of little interest when numerical apertures of .25 or greater are involved. However for much smaller N.A. the spread in radius measurements can easily be noted. This spread is of no optical consequence since it is the limit imposed on any optical system by the Rayleigh criterion.

Using a $10\times$ objective for short radii the

[1] Glazebrook, Sir R. T., editor, *Dictionary of Applied Physics*, Vol. IV (Macmillan and Company Ltd., London, 1922–23), p. 221.

accuracy achievable is better than .025 mm as checked against Twyman[2] interference ring gages. With a Société Genevoise measuring engine and 21× objective N.A. = .50 a 50-mm radius can be measured to approximately .005 mm.

[2] F. Twyman, *Prism and Lens Making* (A. Hilger Ltd., London, 1942), p. 126.

It is of course necessary that the angular aperture of the sphere is sufficiently large so that the focal range is somewhat smaller than the above figures in order to obtain this accuracy.

I wish to thank Mr. H. Ewing, formerly of the Frankford Arsenal Optical Shop, for his observation of the "blackout" on the illuminating prism.

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Heliographic Signaling Mirrors*

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I. INTRODUCTION

A POCKET–SIZE mirror will reflect flashes of sunlight readily visible to an observer ten or twenty miles away on a clear day. Because such a small inexpensive device will thus produce signals visible at considerable distances, all branches of the U. S. armed services now equip their lifeboats, rafts, and floats with heliographic signaling mirrors.

To a passing observer several miles away, signals from a small mirror appear as bright, irregularly spaced flashes of light coming from a point in the distance which is usually near the horizon. The brightness which the flashes appear to have is determined by the size, flatness, and reflectance of the mirror, the distance between rescuer and survivor, and the clearness of the atmosphere. The frequency with which flashes reach the observer is determined by the skill of the survivor and the suitability of the mechanism for aiming mirror-reflected sunlight.

II. MIRROR MATERIALS

The materials from which good shiny mirrors can be made are all familiar, excepting perhaps for mirrorized transparent plastic sheet which has appeared only recently. Signaling mirrors of evaporated aluminum on glass and of precipitated silver on glass have been widely manufactured in spite of the objectionable brittleness of glass. With some mirrors, this objectionable feature has been partially overcome by prior heat treatment of the glass to increase resistance to breakage. With others, the glass has been mounted in a cushioning metal and rubber frame. Coatings of lacquer or paint are, of course, applied to the metal films on these mirrors to prevent corrosion.

Signaling mirrors have also been made of chromium-plated brass and steel. To obtain satisfactory resistance to corrosion from mirrors of this type, the plating must be well applied, or it will not prevent corrosion.

Stainless steel has good resistance to salt-spray corrosion. Although the British made a good signaling mirror of polished stainless steel, it is believed that no American agency tried using this material, in part because it was in short supply during the war, and in part because it did not readily lend itself to the types of construction being tried.

The possibilities of mirrorized transparent plastic are not known. Good plastic mirrors can apparently now be made by both evaporation and chemical precipitation of metal films on transparent sheet. If otherwise practical, plastic signaling mirrors would combine the advantages of light weight with freedom from brittleness. The plastic mirrors examined to date, however, have all tended to warp in warm humid atmospheres till they failed to comply with the experimentally established requirements for mirror-flatness which are described below. Until this

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tendency to warp can be overcome, as it may be in some of the recently developed plastic materials, signaling mirrors of plastic will continue to prove unacceptable.

Mirror reflectance, which at first thought might seem an important consideration in the choice of a mirror material, is actually of secondary importance. Of the materials mentioned above, a clean mirror of silver on glass will reflect more than 90 percent of the light which strikes it at 45°, a mirror of aluminum on glass will reflect about 85 percent, chromium-plated brass or steel 60 to 65 percent, and polished stainless steel, 50 to 65 percent. Although the differences in brightness between flashes from like-sized mirrors of the best and the poorest of these materials could be perceived if the flashes were viewed simultaneously and side by side, the flashes would be of so nearly the same brightness that there would be little difference in their power to attract the attention of a passing observer. Slight bending or warping of a mirror causes a much more serious loss of signal brightness.

III. MIRROR DIMENSIONS AND FLATNESS

Recommended sizes and flatnesses for signaling mirrors were obtained from the results of experiments conducted over an 8-mile range across the city of Washington. Because it is unlikely that a survivor will see any rescue craft more than eight miles away, it was felt that mirrors giving effective signals over this range should be suitable as items of survival equipment.

Flashes of sunlight from a non-flat, 2-inch square, stainless-steel signaling mirror were seen across this 8-mile range. The flashes were not bright, however. On the other hand, flashes of sunlight from every mirror which was 3 by 4 inches in size or larger, and suitably flat, were quite bright.

It may thus be said that a mirror which readily fits into one's pocket is large enough to be an effective signaling mirror. In American survival equipment, mirrors about 3 by 4 inches in size are usually found in life vests and small packs where space and weight are at a premium. Mirrors 4 by 5 inches or 5 by 7 inches are found in lifeboats, in large packs, and where space and weight are not so vital.

Mirrors of these sizes must be nearly flat to be suitable. A perfectly flat signaling mirror will reflect a beam of sunlight into space which has the form of a cone 0.5° in diameter measured from the mirror. This is because the sun is 0.5° in diameter when observed from the earth. If a mirror is not flat, the spread of the beam of reflected sunlight will be greater and the flashes of reflected sunlight will be correspondingly dimmer. In the experiments over the 8-mile range it was found that slight warpage improved a signaling mirror because the additional spread of the reflected beam caused suitably bright flashes of sunlight to reach the observer with greater frequency than from a perfectly flat mirror. A limit was found, however, to the amount of warpage which could occur before the loss of brightness of the reflected flashes became serious.

The sagittal distance of a 4 by 5-inch concave or convex mirror with maximum permissible curvature is about 0.020 inch; that of a 3- by 4-inch mirror with maximum permissible curvature is about 0.010 inch. Although care is necessary to avoid warping the strengthened-glass mirrors during heat treatment and the metal mirrors during mechanical finishing, the required planarity is not difficult to obtain.

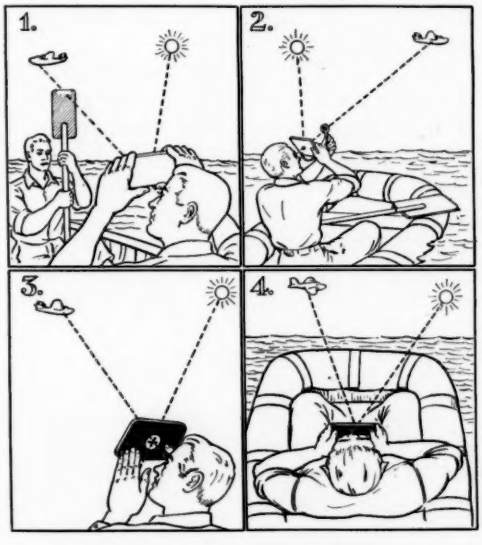

FIG. 1. Four methods of aiming mirror flashes of sunlight: (1) improvised foresight, (2) foresight (British mirror), (3) rearsight (G.E. mirror), and (4) retroreflector.

IV. FORESIGHT-TYPE MIRRORS

Four different methods for aiming mirror-reflected beams of sunlight are illustrated in Fig. 1 in the order of their effectiveness. The methods shown in squares 1 and 2 should both be called foresight methods of aiming a mirror because in each sunlight is reflected from the mirror onto a nearby object in the signaler's line of sight. Square 1 illustrates an improvised method of foresight aiming, and square 2 shows the operation of a foresight signaling mirror built for the purpose.

From the accounts of survivors who have successfully improvised mirror methods of signaling, it appears that, when prepared signaling mirrors are lacking, an improvised method of operation somewhat like that shown in square 1 is most likely to be successful. There is a question whether the particular two-man method of operation shown should be used in a boat or raft on rough water. The motion of the men makes it difficult for them to keep the foresight near the signaler's line of sight to the target. The signaler may be more successful if he uses his own hand as a foresight.

With mirrors manufactured for foresight aiming, correct alignment is assured by sighting through holes. Foresight-type pocket signaling mirrors of practically the same design have been proposed by a Mr. C. H. Wilson of California and a Mr. T. D. Robertson of Australia, and they have been procured and issued as survival equipment by the British armed services.

The British stainless-steel signaling mirror shown in Fig. 2 is 4 by 4 inches in size and $\frac{1}{16}$-inch thick. The key, attached by string to the mirror, is painted white. Rings concentric about the sighting hole have been scratched on the shiny face of the mirror. These enable the signaler

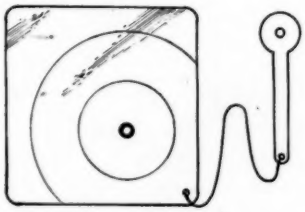

FIG. 2. British 4- by 4-inch stainless-steel signaling mirror with white-painted metal key.

FIG. 3. Rear side of aluminized, tempered-glass signaling mirror manufactured by the General Electric Company and intended for rearsight aiming (courtesy of General Electric Company).

to tell from the shadows of these rings on the white key which way he must turn the mirror to bring the shadow of the dark spot and sighting hole onto the center of the key. The problem confronting a signaler, such as is shown in square 2 of Fig. 1, is to keep his line of sight through the mirror and keyhole aimed toward the target, and at the same time keep the shadow of the sighting hole centered on the keykole.

V. REARSIGHT-TYPE MIRRORS

The rearsight method of aiming a mirror-reflected beam of sunlight makes use of the small pencil of sunlight which comes through the sighting hole. There must be a rearward mirror surface surrounding the sighting hole in addition to the mirror surface facing forward. The pencil of sunlight which comes through the sighting hole is intercepted by some object behind the mirror, such as the signaler's face or hand, and is then observed by reflection in the rearward mirror surface (see Fig. 3 or square 3 of Fig. 1). It is the task of the signaler to turn the mirror so that the reflected image of the intercepted spot of light disappears from the rearward mirror surface into the sighting hole at the same time the line of sight through the hole is directed to the target.

Aiming a mirror by the rearsight method is not a simple task.

The excellent rearsight mirror of Potter's design, including his original specification of tempered glass, was placed in quantity production by Mr. F. C. Perkins of the General Electric Company and widely distributed with American survival equipment. One type is shown in a rear-view photograph, Fig. 3.[1] The mirror is of evaporated aluminum on tempered glass which will not break when dropped from a height of five feet on to a hardwood floor. The sighting hole is in the shape of a cross so that the signaler has a somewhat wider field of view than a round hole would give. To obtain a rearward mirror surface surrounding the sighting hole, the evaporated aluminum film of the mirror was coated with only a clear, tough lacquer. The rear of the mirror with its instructions and the circular reflecting area are shown in Fig. 3.

VI. RETROREFLECTOR TYPE MIRRORS

The idea for the retroreflector type of signaling mirror was submitted to the National Inventors Council for use in the war effort by a Californian named C. H. Learned. He proposed that an aiming mechanism be made by fastening a retroreflector of one of the types widely used in night-visible traffic signs behind a glass or plastic window in a mirror. Whenever sunlight strikes the front of this mirror, that part which passes through the window strikes the retroreflector and starts to return toward the sun. Some of this beam of reversed sunlight is reflected to the rear by the two surfaces of the glass or plastic window. If these two surfaces are parallel to the surface of the mirror, it is easy to see that the direction taken by this rear-reflected beam (usually colored red by the retroreflector) will be exactly opposite to that taken by the main mirror-reflected forward beam. This is shown in the lower part of the drawing, Fig. 4, of the Signal Service Corporation's model of an emergency signaling mirror.

The rearward beam is seen as a red spot by the survivor who places his eye beside the retroreflector in the position shown. This red spot is actually an imperfectly formed image of the sun appearing in space in the direction in which the mirror is reflecting sunlight. In comparison with the foresight and rearsight types of signaling mirror, the retroreflector type of signaling mirror has three important advantages: (1) The method of aiming is not complex; the survivor merely holds his eye behind the mirror, observes the red spot appearing in space, and turns the mirror until this red spot appears to coincide with the target. (2) There is no difficult problem of focusing the eye because the red spot appears at a great distance in front of the mirror. With the other methods of aiming, it is necessary for the survivor to look repeatedly from nearby to a target in the distance and back again. (3) The survivor has a large field of view because the mirror window is fairly large and he can hold his eye immediately behind it.

The Signal Service retroreflector-type signaling mirror shown in Fig. 4 uses an ordinary silver-backed glass mirror built into a cushioning frame and has, as an additional feature, retroreflectors which face rearward for possible use by a survivor in a night rescue. However, the signaling mirror

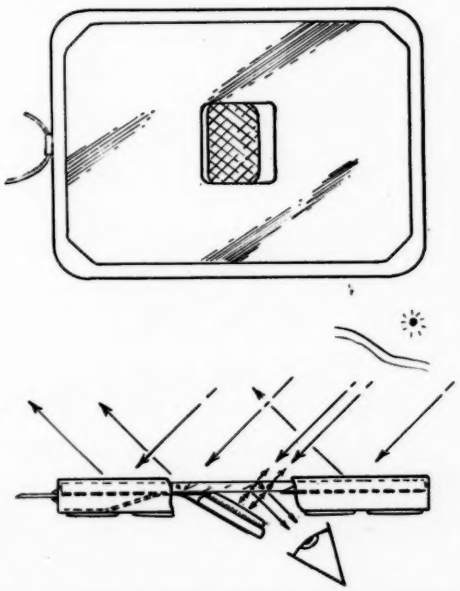

FIG. 4. Retroreflector-type mirror manufactured by the Signal Service Corporation. Part of case is cut away to show directions of rays involved in signaling.

[1] W. M. Potter, "An emergency signaling mirror incorporating high efficiency with precision aiming," Illum. Eng. **39**, 253 (1944); and W. M. Potter, "Solar searchlight," Gen. Elec. Rev. **47**, 7 (1944).

has one disadvantage which it has been possible to eliminate in a recently designed retroreflector-type signaling mirror. The mirror shown in Fig. 4 must be rotated in its own plane till the retroreflector faces roughly toward the sun before the red spot can be seen by looking through the viewing window. The proper positions of reflector, sun, and signaler's eye are indicated in Fig. 4 and in square 4 of Fig. 1.

A retroreflective material manufactured by the Minnesota Mining and Manufacturing Company comes in the form of sheets to which small spherical glass beads have been affixed. It goes under the brand name of Scotchlite. One form of this material is well suited for making retroreflector-type signaling mirrors. It can be cut into washers with sighting holes in the center and each washer can then be mounted behind a round window of glass or plastic in a mirror. When the retroreflector has this washer shape, it makes no difference how the mirror is turned in its own plane; the necessary rearward beam to form the red spot will always come back through the sighting hole. The drawing of a Scotchlite type of signaling mirror which the General Electric Company has made with an aluminum-coated, tempered-glass mirror, and disk-shaped protecting cover glass is shown in Fig. 5. This improved signaling mirror had not reached quantity production at the time the war ended.

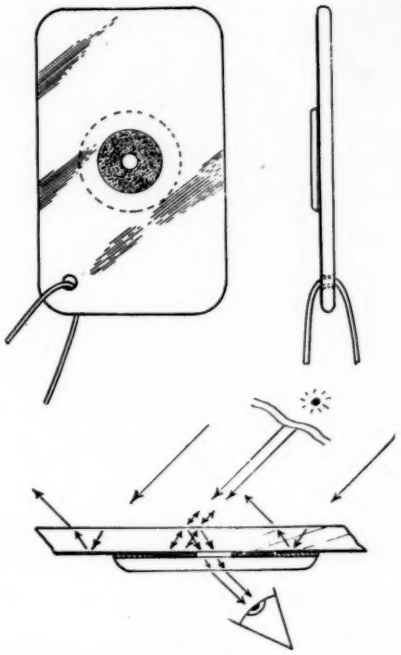

FIG. 5. Retroreflector-type mirror made with Scotchlite washer behind round window in tempered-glass mirror by the General Electric Company. Cross section of aiming mechanism below shows directions of rays involved in signaling.

VII. COMPARATIVE TEST OF SIGNALING MIRRORS

In order to compare some of the above types of mirrors for effectiveness under simulated survival conditions, a series of tests were conducted in May, 1944 at the U. S. Coast Guard Air Station, Elizabeth City, North Carolina. The mirrors compared were operated by the four methods illustrated in Fig. 1. Unfortunately the Scotchlite type of retroreflector mirror shown in Fig. 5 had not been developed at the time the tests were made.

To operate the signaling mirrors, six subjects with no previous knowledge of these devices were chosen from among the machinists at the station. The subjects were taken in a radio-equipped cabin cruiser to an area in Albermarle Sound where the water was somewhat choppy, and there a 4-man rubber raft was tied to the stern of the anchored boat. A scouting plane repeatedly passed around the boat and raft at an elevation of about 500 feet. It always approached from the up-sun direction, circled two miles from the raft, and then left to go six miles up-sun before turning for the next run. The small scouting plane could be seen only when it was within about four miles of the boat. During each circuit, subjects in the raft tried to signal with first one and then another of the mirrors.

An observer in the scouting plane reported by radio the number of distinct flashes of sunlight which he saw during each 30-second period. A comparison of two mirrors was made during each run so that valid comparisons would be obtained even though changes in the subject's ability or the weather might change the rating scale during the course of observations. Fortunately the average rate at which flashes were received from any one mirror did not change markedly, either with changes of subjects or with course of the experiment. The results of the test can be summarized by giving the average rate at which flashes of

sunlight were received by the plane from each signaling mirror:

	Average number of flashes observed per minute
1. Improvised foresight aiming method	0.3
2. Stainless-steel foresight mirror	8.
3. Tempered-glass, rearsight (G.E.) mirror	14.
4. Retroreflector (Signal Service Corporation) mirror	35.

From the results it is apparent that the retroreflector-type mirror performed most effectively, while the improvised foresight method was poorest. The stainless-steel foresight and the G.E. rearsight mirrors were intermediate in effectiveness.

VIII. CONCLUSION

The heliographic signaling mirror is a small, inexpensive, yet effective signaling device; however, it has limitations. Naturally it will not work when the sun is behind clouds. In addition, it cannot be used to direct signals at angles too far from the sun. Each of the methods of aiming described above gets more difficult to use as the angle between the sun and target gets larger than 90°. The retroreflector-type mirrors will not work at angles greater than about 135° because the retroreflective materials cease to function with light incident on them at more than 65 or 70° from normal. Even though flashes cannot be aimed in directions more than 135° from the sun, signals can nevertheless be directed to every part of the sky when the sun is more than 45° above the horizon, and they can be sent to all but a small segment of the sky when the sun is lower.

With retroreflector-type mirrors, there is a cone of directions immediately surrounding the sun into which signals cannot be aimed. The existence of this cone of directions is due to the necessity for lateral travel of the aiming beam within the mirror from the retroreflector where it starts to the sighting hole (see Figs. 4 and 5). When sunlight strikes a mirror perpendicularly, there is no lateral travel of this beam. In practice, this cone of directions in which the aiming mechanism does not operate is from 20 to 30° in diameter with the sun as its center. However, it covers only a small fraction of the total area of the sky, and the user of a signaling mirror can easily hold this area to a minimum by remembering to move his eye to the side of the sighting hole opposite the sun and nearest the target.

In spite of limitations, the new retroreflector-type signaling mirror seems well worth placing on all American ships and overseas aircraft along with the other best items of survival equipment developed during the war. It may be expected that the new device will also find use by surveyors, mountain climbers, and other persons engaged in outdoor activities. Forest Service officials have expressed interest in the mirror as a possible device to use in signaling between airplanes and crews on the ground.

To enlarge its possible usefulness and perhaps to improve it as an air-sea rescue device, a method has been developed to obtain red, yellow, and green signals from a signaling mirror which are readily distinguishable from each other and from the usual achromatic signals. To obtain each of these chromatic signals, a film of red, yellow, or green cellulose acetate film was held in front of the mirror. A hole was cut in each film to coincide with the aiming window. This film was purposely held so that its surface was not parallel to the mirror surface because it was found that the achromatic reflection from the front face of a film could seriously dilute the color of the signal reflected by the mirror. By making the film nonparallel to the mirror, the beam reflected from the front surface of the film was thrown harmlessly to one side of the aimed beam.

There is a question whether the best signaling mirror for survival use is one which gives a distincly chromatic (red or orange) signal or one which gives a plain achromatic signal. A red or orange signal is more likely to attract the attention of the casual passer-by than an achromatic signal. However, the greater complexity of the device which gives the colored signal may offset the advantage of color. In general, apparatus for survival use should be as simple and foolproof as possible.

Calibration of a Set of Master Wedges

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Frankford Arsenal, Philadelphia, Pennsylvania
(Received November 9, 1945)

THERE are numerous precision methods available for the comparison of the light deviation of wedge prisms. The present paper deals with two precision methods for the absolute calibration of wedge prisms. It seems desirable to describe these methods in detail because of the accuracy attainable with precision apparatus. Method II allows determinations to be made to better than 0.25″ of arc in a very simple fashion.

METHOD I. "TELESCOPE AND SCALE"

The wedge is placed before the objective of a high power telescope and based on a scale set at a known distance. A reading is taken on the scale and the wedge rotated through 180° and a second reading taken. The angle subtended at the wedge is $2D$ or twice the light deviation of the wedge.

A schematic diagram of the optical system is shown in Fig. 1. The objective lens L_1 was of 5 inches aperture and 60 inches focal length. In making measurements the telescope objective was stopped down to 2.5 inches the aperture of the wedges. The eyepiece of the telescope was a 30-power tool maker's microscope M provided with a filar micrometer eyepiece to enable the accurate measurement of fractions of a millimeter. A standard meter bar S was used as the scale on which readings were taken. This meter bar was provided with a Bureau of Standards Calibration. The meter bar was illuminated by means of the light from a mercury arc.

PROCEDURE

The telescope objective was "squared on," making use of an artificial star placed near the center of the meter bar. After "squaring on" the objective, the meter bar was made normal to the optic axis of the telescope by means of an auxiliary mirror placed in contact with the meter bar. Plumb lines were dropped to the floor from the end of the meter bar and also from the position of the wedge rear surface with proper offsetting. The distance K was chained with a standard calibrated steel tape. K was found to be 30989 mm ± 3 mm. One millimeter displacement on the scale corresponded to 6.65604 seconds of arc. The probable error in determining the double deflection on the meter bar was less than ±.02 mm. This error would give rise to an uncertainty in the light deviation of ±.07″. It is necessary to know the light deviation of these wedges for the D lines of sodium. Because of the requirement of intense illumination when using the high power telescope, it was not possible to use a sodium lamp to illuminate the meter bar. Use of mercury arc illumination requires a correction for wave-length which was made as follows. The wedges were all made from Borosilicate Crown Glass $N_D = 1.51700$. It was assumed that the wave-length effective with the mercury arc was such as to yield an index of refraction half-way between the yellow and green mercury lines, i.e., 1.51825. The calculated correction to correct observed readings to N_D is −.145″ per minute light deviation. This correction has been applied to results recorded in Table I. An analysis of the errors involved in making the determination of the light deviation by the telescope and scale method as described above makes it appear that the values given in Table I under (Tel. and Scale) do not depart from the absolutely correct value of D by more than ±.25 seconds of arc. The 10′, 20′, and 30′ wedges were not measured by the telescope and scale method since the chromatic correction would be too large to allow the achievement of the desirable precision.

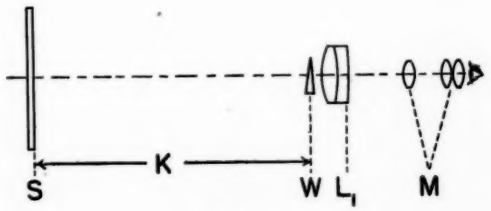

FIG. 1.

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MULTIPLE BEAM METHOD

If parallel light is incident normally on a wedge of glass both sides of which are partially coated with a reflecting film, one will see a number of images due to the multiple reflections between the faces of the wedge. A drawing is given in Fig. 2 showing the geometrical consequences of the multiple reflections. The angle R_k between the normal to the emergent surface and the kth refracted beam will be given by the equation

$$\sin R_k = n \sin (2k-1)A, \qquad (1)$$

where n is the index of refraction and A the refracting angle. It can be seen that if $I \leqq 0$, $dR_k = dI$ if dI is small. In any case the error introduced into A due to an uncertainty in dI will be

$$dA = AdI/[n(2k-1)] \text{ (approximately).} \qquad (2)$$

Differentiating Eq. (1) with respect to n the error introduced into the measured value of A due to an error in determining n is

$$dA_n = \frac{-dn}{n(2k-1)\cot(2k-1)A}. \qquad (3)$$

The error introduced into the light deviation due to an error in n will be

$$dD = (n-1)dA_n + Adn. \qquad (4)$$

Necessary Flatness of Surfaces

If the true focus of the objective does not depart from the position of the cross wires by more than the focal range $\Delta F = 4\lambda f^2$ (where λ is the wave-length of light used and f is the ratio of focal length to aperture of objective), no appreciable parallax can be detected. The above-mentioned equation for ΔF arises from the application of the quarter-wave path error condition to the lens. The application of the quarter-wave criterion to the wedge when multiple reflections occur leads to

$$\frac{S}{\lambda} = \frac{1}{8(2k-1)(n-1)}, \qquad (5)$$

where S/λ is the allowable path error expressed in wave-lengths.

TABLE I.

Nom. D	N_D	A-mult. beam	D-tel.-scale	D-lines D-mult. beam	D-green (calc)
15″	1.51727	*I*28.48″	14.59″	*I*-C14.73″	*I*14.76″
30″	1.51734	*I*58.92″	30.58″	*I*-C30.48″	*I*30.55″
1′	1.51722	117.02″	60.37″	60.53″	60.76″
2′	1.51722	229.35″	118.91″	118.62″	119.08″
3′	1.51727	346.04″	179.13″	179.00″	179.70″
4′	1.51751	467.46″	241.79″	241.92″	242.85″
5′	1.51684	584.33″	302.02″	302.01″	303.17″
10′	1.51685	1165.39″	*S*602.7″	602.36″	604.68″
20′	1.51714	2317.12″	*S*1198.9″	1199.39″	1203.03″
30′	1.51714	3474.55″	*S*1797.4″	1797.15″	1804.11″

S = direct spectrometer measurement.
I = direct interferometer measurement.

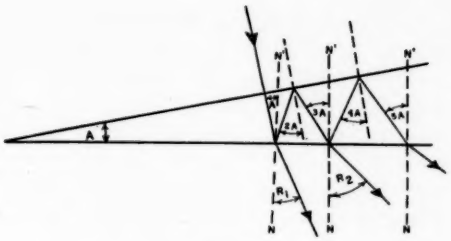

FIG. 2.

EXPERIMENTAL PROCEDURE

Measurement of the refracting angle by the method of multiple images requires the measurement of the angle R_k to high precision. The instrument chosen to make these measurements was a Gaertner Precision spectrometer whose divided circle could be read to 1 second of arc making use of four reading microscopes. The observing telescope on this instrument was 20 power. The original lenses on the spectrometer were tested and both found to have path errors in excess of $\frac{3}{4}\lambda$. New lenses, "cemented doublets," were designed and manufactured and tried out after cementing; they were tested on the Twyman lens interferometer. The interference pattern could be made to go "all black" over the entire surface of the lens showing a path error of the order of $\frac{1}{10}\lambda$ or less. After installing the new lenses in the spectrometer, tests showed that parallax was still present in the instrument due to aberrations in the eyepiece. However the parallax was small and would cause not the slightest error for an experienced observer, since with no reasonably small shift of the observer's eye was parallax detectable.

After proper leveling and focusing of the telescope and collimator of the spectrometer, a direct

reading was taken setting on the slit which was illuminated with light from a sodium vapor lamp. One of the half aluminized wedges was then placed in a holder and based by rotation observing one of the higher order reflected and transmitted images. The telescope of the spectrometer was then offset by an angle slightly greater than the angle A of the wedge which had preliminarily been determined to a second or better by a direct measurement of the light deviation. The prism table of the spectrometer was rotated about its axis and leveled so as to autocollimate* the rear face of the wedge into the spectrometer telescope by means of a Gauss eyepiece. This adjustment permits the light from the collimator to impinge nearly normally on the entrance face of the wedge. A reading was then taken on the circle and the telescope rotated to the kth image and thus the angle R_k is measured. The value thus obtained for R_k is in error due to the fact that $dI \neq 0$. However the correction to be applied is as follows. Offset angle minus angle $A = dI$ and since dI is small and since $dI = -dR_k$ to a high degree of approximation, true R_k = measured $R_k - dI$. When the true value of R_k has been measured, A can be determined by the use of Eq. (1) if n is known. The light deviation of the wedge for normal incidence can now be calculated making use of the well-known exact equation

$$\tan A = \frac{\sin D}{n - \cos D}. \qquad (6)$$

Since D is small and due to the small sensitivity of the cosine function in the neighborhood of zero angle it is not necessary to solve the above quadratic equation for $\cos D$. A sufficiently precise value of $\cos D$ can be obtained making use of the approximate well-known equation

$$D = (n-1)A. \qquad (7)$$

The insertion of $\cos D$ obtained by means of Eq. (7) into Eq. (6) yields the final value of D to sufficiently high precision for this work.

QUALITY OF SURFACES

Equation (5) gives the necessary flatness of surface required. For use of the 7th image a surface quality of about $1/50\lambda$ is demanded. The surfaces on these wedges were all good to somewhere between 1/20 wave or better over the aperture actually used.

MEASUREMENT OF INDEX OF REFRACTION

Index of refraction was measured using a Bausch & Lomb precision refractometer which was calibrated by means of a B.S.C. prism measured by the National Bureau of Standards and stated to have an index of 1.51665. The index was measured for the mean wave-length of D_1 and D_2 lines of sodium. From Table I it can be seen that there are several different indices present in this set of wedges. This situation furnishes a satisfying check on index determinations. On interchanging wedges in making a number of determinations of index, repeating values to 1 and very often no difference in the 5th decimal, gives a good indication of absolute as well as relative accuracy in view of the small extrapolation from the standard prism. It is thought that the indices given in Table I are accurate to one or at most two in the fifth decimal place.

DISCUSSION OF ERRORS

Sources of error:

(1) Measurement of R_k
(2) Determination of dI
(3) Measurement of index of refraction.

(1) From a summary of measurements made on R_k the probable maximum error in R_k does not exceed ± 1.5 seconds.
(2) The probable maximum error in dI should be identical with (1) above ± 1.5 seconds.

* Setting the normal with the Gauss eyepiece was done in the following fashion. Instead of setting the wedge so that the image of the cross hairs was completely hid by the real cross hairs a small vertical displacement of the reflected image was introduced. The appearance in the eyepiece was as shown below.

The angular setting was made so that the fine cracks of light showing between the cross hairs and their image had the same size in all quadrants. Making use of this artifice allows a setting to be made more precisely than when a "hiding" method is used.

(3) The index of refraction is believed to be known to 1 in the 5th decimal place.
(1) Gives rise to an error of $\pm.04''$ in D
(2) Gives rise to an error of $\pm.04''$ in D
(3) Gives rise to an error of $\pm.022''$ for a 30′ wedge.

The total error maximum might be the sum of (1)–(2) which could be $\pm.10''$

The total error in A will be twice as great and may amount to $\pm.20''$

COMPARISON OF WEDGES

(1) Twyman Interferometer

The method of comparing wedges with the Twyman interferometer in which the wedge is based and the number of fringes crossing the aperture counted is well known. This method of counting and estimating fractions of a fringe is subject to considerable uncertainty. A somewhat more refined and accurate procedure, which is obvious, will be described.

Without anything in the optical path of the interferometer the rear mirror is set in nearly the parallel position so that several bands cross the aperture. The mirror is oriented so that fringes are vertical. A master wedge is placed in the light path of the interferometer and rotated so that the base apex line of the wedge is parallel to the base of the interferometer. The wedge to be compared is inserted in series with the master with its base opposed to that of the master wedge. The adjustable mirror is oriented so that about 10 fringes cross the field of view and the orientation is such that the fringes are vertical. The angle at which the mirror now stands from its true parallel position is determined as follows. A micrometer caliper is placed in the parallel beam in front of the viewing lens. The jaws of the caliper are set so as to fall in the middle of two black bands near the extremes of the aperture. The even number of bands traversing the distance between micrometer jaws are then counted. If N is the number of fringes counted, A is the micrometer reading, and λ the wave-length, the angular inclination of the two interfering beams will be θ expressed in radians

$$N\lambda/A=\theta_1.$$

For green mercury light this equation can be written

$$4.4346(N/A)=\theta_1,$$

where θ_1 is expressed in seconds of arc and A is the aperture in inches. The master wedge and wedge being measured are rotated in such a manner so as to interchange base and apex positions for both wedges. θ_2 is now determined in the same manner as θ_1 formerly. Once θ_1 and θ_2 have been determined, ΔD the difference in light deviation of the two wedges, is given by the equation,

$$(\theta_1-\theta_2)/4=\Delta D,$$

where the above value $\theta_1-\theta_2$ is the algebraic difference of these two quantities.

The author wishes to acknowledge his indebtedness to Mr. O. K. Kaspereit and Mr. F. B. Patrick for the use of the precision spectrometer and their design of the new spectrometer lenses. The author is indebted to Mr. R. A. Boydston for coating the partially reflecting surfaces, to Mr. J. Kiska for his assistance in measurement of the index of refraction, and finally to Mr. H. Hill for the production of an excellent figure on the surfaces of these wedges.

Book Review

Collected Papers on Metallurgical Analysis by the Spectrograph

D. M. SMITH, *Editor*. Pp. 162+x. British Non-Ferrous Metals Research Association, London, 1945. Price 21s.

The papers in this collection, selected from those released as Research Reports by the Association, deal exclusively with applications of spectrographic analytical methods to non-ferrous metallurgy. An excellent summary of recommendations for the processing of photographic plates in spectrographic analysis is followed by series of papers dealing, respectively, with detailed methods of analysis of aluminum and its alloys, of lead and its alloys, of zinc and of copper alloys, and of platinum. Numerous working curves are given together with tabular data which will be found of great value to the metallurgical spectrographer. The collection is provided with an excellent index.

G. R. HARRISON
Massachusetts Institute of Technology

Letters to the Editor

Units in the Trichromatic System

PARRY MOON
Massachusetts Institute of Technology, Cambridge, Massachusetts
AND DOMINA EBERLE SPENCER
Tufts College, Medford, Massachusetts
December 3, 1945

IN this Journal of June 1945,[1] we suggested the names *munsell*, *young*, and *ostwald* for the m.k.s. units of the trichromatic quantities X, Y, and Z. A number of people have objected to the names *munsell* and *ostwald* as being inappropriate in a trichromatic system, and Dr. Deane B. Judd has kindly suggested alternatives. In the light of these developments, we should like to modify our previous suggestion.

The first question to be decided is whether it is advisable to have *any* names for the units of X, Y, and Z. Ordinarily, colorimetrists do not take the trouble to express X, Y, and Z in absolute units, since they employ these quantities only as "catalytic agents" in obtaining the homogeneous coordinates x and y. This fact probably accounts for the lack of previous suggestions regarding trichromatic units. But the trichromatic system is no longer limited to the specification of reflection from surfaces. As the system expands in usefulness and is applied to the specification of lamps and daylight and to other problems of illuminating engineering, the need for units becomes more evident.

The quantities X, Y, and Z are defined by the equations,

$$\left.\begin{aligned} X &= \int_0^\infty \bar{x}(\lambda)J(\lambda)d\lambda \\ Y &= \int_0^\infty \bar{y}(\lambda)J(\lambda)d\lambda \\ Z &= \int_0^\infty \bar{z}(\lambda)J(\lambda)d\lambda \end{aligned}\right\}, \qquad (1)$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the internationally accepted C.I.E. weighting functions and $J(\lambda)$ is in watts per square meter, per micron wave-length band. Each equation may be regarded as defining a new concept, distinct from radiant energy. The second equation is employed extensively in photometry and thus there can be no question about the necessity of a unit for Y. The need for units of X and Z is not so striking because the colorimetric specification of radiation is not as widely used as the photometric specification.

Whenever the trichromatic system is applied to light sources, however, the need manifests itself. In the consideration of Planckian radiation, for example, the total watts per square meter of radiating surface may be obtained from the Stefan-Boltzmann relation. Of even greater importance in many applications is the trichromatic specification obtained from Eq. (1). The result is in absolute units, but *what* absolute units? One may say "watts/m² evaluated with respect to the C.I.E. weighting functions," but that is hardly a convenient expression. A similar need occurs in the specification of radiation from commercially available light sources. Spectral distribution curves, expressed in watts/m² per micron received at a given distance from the source, are obtained by means of a spectroradiometer. These curves are evaluated according to Eq. (1) and give X, Y, Z for a receiving surface at the given distance. Another need arises in the specification of sunlight and skylight at the earth's surface. In all these cases, a numerical specification of X, Y, and Z is needed. Furthermore, one must somehow indicate that the results are not in watts per unit area but are in new units whose size depends on the unit of $J(\lambda)$ and on any conversion factors that may have been inserted in Eq. (1). Thus there is a practical need for units of X, Y, Z; and it is highly probable that this need will continue to grow as the field of colorimetry expands.

The next question deals with the names to be used for the units. Our original idea was merely to choose names of investigators who worked with color. The name *young* seemed particularly appropriate because of Thomas Young's pioneer work on the trichromatic theory of vision. We have heard of no objection to this name.

Dr. Judd has advocated names more closely associated with the trichromatic system than are *munsell* and *ostwald*. He has suggested *könig* and *ives*, or *könig* and *priest*. A. König, H. E. Ives, and I. G. Priest have all done notable work in the development of the trichromatic system. The difficulty in the formation of the plural *ives* leads us to favor the word *priest*. We are happy, therefore, to amend our previous suggestion and to advocate the expression of X in königs, Y in youngs, and Z in priests. For ease in distinguishing the names, note that alphabetically $k \rightarrow p$ and $X \rightarrow Z$.

We hope that other members of the Society will express their opinions on the subject so that the best solution can ultimately be obtained.

[1] P. Moon and D. E. Spencer, J. Opt. Soc. Am. **35**, 399 (1945).

Letter to the Editor

DEANE B. JUDD
National Bureau of Standards, Washington, D. C.
December 17, 1945

THIS will refer to a note to the Editor by Moon and Spencer entitled "Units in the Trichromatic System," copy of which they very kindly have sent to me for comment.

In my writing on colorimetric subjects I have not found it necessary to express X, Y, and Z in absolute units; instead I make use of the chromaticity coordinates x and y. I would, therefore, cast my vote in favor of not adopting any units or any names for X and Z at all. The luminous quantity Y may refer to luminous reflectance, luminous transmittance, illuminance, or luminance. The unit proposed by Moon and Spencer, to be known as the "Young," refers only to illuminance. Since there are already two well-established units for illuminance, the lux or lumen per square meter or metercandle, and the lumen per square foot or footcandle, it would, it seems to me, be piling confusion on confusion to have another unit. I am therefore in favor of none of the proposed units.

My suggestion of the names, König, Ives, and Priest, should therefore not be construed as a vote in favor of giving names to these units. I made the suggestion because I think that, in the unlikely event that colorimetric practice should develop a need for new absolute units of X, Y, and Z, such names would be much more appropriate than those originally proposed.